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Numerical Estimation in Preschoolers

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Children's sense of numbers before formal education is thought to rely on an approximate number system based on logarithmically compressed analog magnitudes that increases in resolution throughout childhood. School-age children performing a numerical estimation task have been shown to increasingly rely on a formally appropriate, linear representation and decrease their use of an intuitive, logarithmic one. We investigated the development of numerical estimation in a younger population (3.5- to 6.5-year-olds) using 0–100 and 2 novel sets of 1–10 and 1–20 number lines. Children's estimates shifted from logarithmic to linear in the small number range, whereas they became more accurate but increasingly logarithmic on the larger interval. Estimation accuracy was correlated with knowledge of Arabic numerals and numerical order. These results suggest that the development of numerical estimation is built on a logarithmic coding of numbers—the hallmark of the approximate number system—and is subsequently shaped by the acquisition of cultural practices with numbers.

Keywords: numerical development, number representation, numerical estimation, number line

It is widely believed that numeracy is founded upon an early nonsymbolic system of numerical representation (for reviews, see Carey, 2001; Feigenson, Dehaene, & Spelke, 2004). Indeed, children show numerical abilities long before language acquisition and formal education. The ability to discriminate between two numerosities improves from a ratio of 1:2 to 2:3 before the first year of life (Lipton & Spelke, 2003; Xu, Spelke, & Goddard, 2005). For example, 6-month-olds can discriminate between 16 and 8 dots but not between 16 and 12 dots, whereas 9-month-olds discriminate both ratios. Numerical acuity is progressively refined throughout childhood, attaining adult's ability in early adolescence (Halberda & Feigenson, 2008). With language acquisition, children understand how number words map onto distinct numerosities (Wynn, 1996), first in their counting range and then also outside of it (Lipton & Spelke, 2006).

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It has been proposed that the infant's sense of numbers is based on two "core systems" (Feigenson et al., 2004): (a) a *small number system* accurate for numerosities up to 3, essentially a perceptual system for tracking objects, and (b) an *approximate* number system for representing larger numerosities. The latter encodes numerosities as analog magnitudes, usually thought of as overlapping distributions of activations on a logarithmically compressed mental number line (see Dehaene, Piazza, Pinel, & Cohen, 2003, for review). Its precision increases throughout childhood (Halberda & Feigenson, 2008) and is correlated to math achievement at different ages (Halberda, Mazzocco, & Feigenson, 2008).

Children in our culture achieve a mental representation of number that goes beyond these core number systems in at least two different ways. First, numerate children and adults are able to go beyond approximate numerosities and can distinguish and represent exact numerosities greater than 3. Second, the concept of number in mathematically educated adults implies a linear (rather than logarithmic) mapping between numbers and space (Siegler & Opfer, 2003; Zorzi, Priftis, & Umiltà, 2002), such that numbers can be used for measurement. It is a controversial matter as to how both advances are achieved, although it is clear that the child's experience with counting and number words plays a major role (e.g., Le Corre & Carey, 2007). A developmental transition from logarithmic to linear numerical estimation has been documented in the seminal studies of Siegler and collaborators suggesting that children's representation of numbers changes over time with increasing formal knowledge (Siegler & Booth, 2004; Siegler & Opfer, 2003).

Siegler and Opfer (2003) investigated estimation abilities of primary school pupils and adults with the number-to-position task where numbers had to be placed on number lines with 0 at one end and 100 or 1,000 at the other. This task requires translation between numerical and spatial representations without assuming knowledge of specific measurement units. Children's estimates changed with age and shifted from a logarithmic to a linear representation. At Grades 2 and 4 they overestimated small numbers and compressed large numbers to the end of the scale (logarithmic positioning) when the context was unfamiliar (0–1,000) but positioned numbers linearly in a familiar context (0–100). In contrast, Grade 6 children positioned numbers linearly on both small and large scales, just like adults. Interestingly, younger children treated the same numbers differently—identical numbers being placed linearly or logarithmically according to the interval of reference—indicating how the context influenced the numerical representation deployed in the task and how the choice among these multiple representations is dependent of age and experience.

Siegler and Booth (2004) replicated the study with a population of preschoolers, first graders, and second graders (mean ages: 5.8, 6.9, and 7.8 years, respectively) and observed the same developmental sequence for the 0–100 number line. Mathematical achievement was found to correlate with the linearity of the estimates highlighting how formal knowledge modulates the number–space mapping. Booth and Siegler (2006) found a correlation between performance in the number-to-position task and other numerical estimation tasks such as approximate addition, numerosity, and measurement estimation, suggesting that all of these numerical approximation tasks tap onto a common internal representation.

The aim of the present study was to assess children's ability to provide reliable estimates (whether logarithmic or linear) as early as 3.5 years old and to further characterize the developmental trend that leads to the emergence of a linear and formal representation of numbers. The shift from logarithmic to linear positioning is firmly established in the context of 0–100 or 0–1,000 number lines, but not when the number range is restricted to units or teens. Indeed, contrary to the hypothesis that logarithmic positioning is mandatory prior to a linear one even in the small number range, Whyte and Bull (2008) observed linear positioning on a 0–10 number line (also used by Petitto, 1990) in their intervention study on a group of young preschoolers (mean age 3.8), both at the group level and in about 75% of individual children.

In Experiment 1, we administered the 0–100 interval used by Siegler and Booth (2004) as well as a smaller 1–10 interval. Numbers in the small interval should be familiar to the youngest children. Zero was excluded (cf. Petitto, 1990; Whyte & Bull, 2008) because children learn the counting sequence from one and its concept is usually introduced later (Butterworth, 1999). The use of the 0–100 interval allowed us to confirm that the oldest children in our study had comparable performance to the youngest group tested by Siegler and Booth. In Experiment 2, we tested a much larger sample of children and we replaced the larger interval with a new 1–20 interval to further investigate the developmental pattern within a smaller range of numbers. Observing a shift from logarithmic to linear with 1–10 or 1–20 intervals would demonstrate the deployment of logarithmic representations even within the small number range. Children were also tested on basic numerical knowledge to investigate its relation to their ability to estimate. This was done with a simple digit naming task in Experiment 1, which, for very young children, has been found to be a strong predictor of other numerical tasks (Ho & Fuson, 1998; Huntley-Fenner & Cannon, 2000). In Experiment 2, children had to order by magnitude collections of dots and Arabic numbers and had to recite the counting sequence up to 10 on their fingers.

We predicted that the youngest children, considering the absence of formal education,[1] would reveal a purely logarithmic representation even in the small number range and that the developmental pattern would show a shift from logarithmic to linear positioning, but only for the smaller intervals. At around 4.5 years children start understanding how number words beyond four map onto sets of items and start to understand the counting principles (i.e., one-to-one correspondence and cardinality; Gelman, 1978; Le Corre & Carey, 2007), which might play a role in the transition between representations. Thus, the hypothesis that children possess multiple ways of representing numbers (Siegler & Opfer, 2003) leads to the prediction that performance might become linear on the smaller interval but remain logarithmic on the larger interval.

Experiment 1

Method

Participants. Forty-six children (21 girls) were recruited in two different kindergarten schools from northeastern Italy. They were all from the Italian ethnic group and mostly of middle socioeconomic status. Children were divided in three groups according to age: the youngest group ($n = 11$) had a mean age of 48 months (range: 42–53), the middle group ($n = 16$) had a mean age of 60 months (range: 54–64), and the oldest group ($n = 19$) had a mean age of 71 months (range: 65–75).

Procedure. Two trained female teachers, one from each school, met individually with the children during school hours in a quiet classroom. Children were first tested on a digit naming task. Arabic digits from 0 to 9, randomly presented on separate cardboards (5 cm^2), had to be named aloud without receiving feedback (score: 1 point for each correct item).

In the number-to-position task, children were presented with 25-cm long lines in the center of white A4 sheets. Two different intervals were administered: 1–10 and 0–100. The ends of the lines were labeled on the left by either *1* or *0* and on the right by either *10* or *100*. The number to be positioned was shown in the upper left corner of the sheet. All numbers except for 1, 5, and 10 had to be positioned on the smaller interval, whereas for the larger interval numbers were 2, 3, 4, 6, 18, 25, 48, 67, 71, and 86 (corresponding to Sets A and B for the same interval in Siegler & Opfer, 2003).

[1] In the Italian educational system, a child may start preschool up to 3 years prior to Grade 1. Enrolment to preschool is still an optional choice for parents as well as the number of years of attendance. For this reason, the Ministry of Education does not give strict directives on the topics, goals, and knowledge to reach by the end of preschool. Teachers are invited to introduce numerical concepts by creating numerical experiences usually by using games and songs. The most common practice is to use songs to enumerate sets and to teach children how to read digits 1–9. Moreover, classes are not always subdivided by age or number of years of preschool completed, giving the opportunity for more experienced children to help younger ones or newcomers.

The order of presentation of the two intervals and the order of items within each interval were randomized. Each line was seen separately from the others. The instructions were given as follows:

> We will now play a game with number lines. Look at this page, you see there is a line drawn here. I want you to tell me where some numbers are on this line. When you have decided where the number I will tell you has to be, I want you to make a mark with your pencil on this line.

To ensure that the child was well aware of the interval size, the experimenter would point to each item on the sheet while repeating for each item: “This line goes from 1 [0] to 10 [100]. If here is 1 [0] and here is 10 [100], where would you position 5 [50]?” The experimenter always named the numbers to place. Numbers 5 and 50 were used as practice trials for the small and large intervals, respectively. No feedback was given. Experimenters were allowed to rephrase the instructions as many times as needed without making suggestions about where to place the mark.

Results

Children’s estimation accuracy was computed as percentage of absolute error (*PE*). This was calculated with the following equation (Siegler & Booth, 2004):

$$PE = \left| \frac{\text{estimate} - \text{target number}}{\text{scale of estimates}} \right| \times 100.$$

For example, if the estimated position of 45 on the 0–100 interval corresponds to 60, the *PE* would be 15%; that is, $[(60 - 45)/100] \times 100$.

A one-way analysis of variance (ANOVA) on mean *PE* was computed for each interval, with age as between-subjects factor. For both intervals, results indicated that the three groups were significantly different and the accuracy of estimation increased with age (1–10 interval: $F[2, 43] = 6.14$, $p < .01$, $\eta^2 = .22$; 0–100 interval: $F[2, 43] = 4.22$, $p < .05$, $\eta^2 = .16$; henceforth all *p* values are two-tailed unless otherwise noted). The youngest and the middle groups significantly differed from the oldest group on post hoc comparisons for the interval 1–10 (*p*s < .05). *PE*s for the youngest, middle, and oldest group were 28%, 24%, and 15%, respectively. For interval 0–100, *PE*s were 32%, 30%, and 23% (from youngest to oldest, respectively), and only the youngest group significantly differed from the oldest group on post hoc comparisons ($p < .05$). It is worth noting that for the 0–100 interval the accuracy of estimation for our oldest group is slightly better than the accuracy of the comparable age group (5.8 years old) studied by Siegler and Booth (2004; 27% in their study).

Fits of linear and logarithmic functions were computed to analyze the pattern of estimates. Following Siegler and Opfer (2003), we computed these fits first on group medians and then for each individual child.

Group analysis. For group medians (see Figure 1), the difference between models was tested with a paired-sample *t* test on the absolute distances between children’s median estimate for each

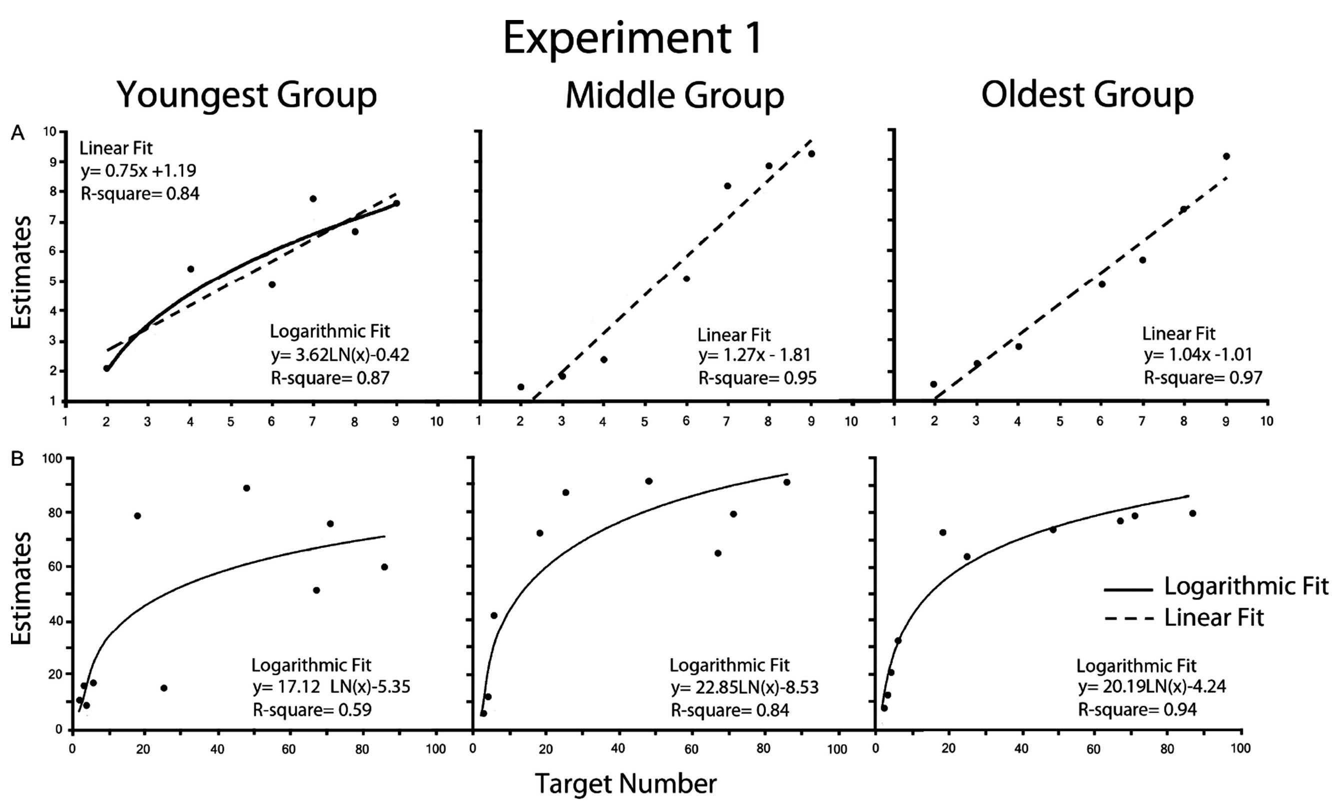

Figure 1. Children estimates and best fitting models in Experiment 1 as a function of age group for 1–10 (A) and 0–100 (B) number lines.

number and (a) the predicted values according to the best linear model and (b) the predicted values according to the best logarithmic model. If the t test indicated a significant difference between the two distances, the best fitting model was attributed to the group. For the 1–10 interval, the model with the highest R^2 was logarithmic for the youngest group (R^2 log = 87%, $p < .01$), but it did not significantly differ from the linear fit (R^2 lin = 84%, $p < .01$), $t(6) = -1.17$, $p > .05$. For the two older groups, the fit of the linear model was significantly better than the fit of the logarithmic model (intermediate group: R^2 lin = 95%, $p < .001$, vs. R^2 log = 89%, $p < .01$, $t[6] = 2.82$, $p < .05$; oldest group: R^2 lin = 97%, $p < .001$, vs. R^2 log = 88%, $p < .01$, $t[6] = 4.05$, $p < .01$). For the 0–100 interval, the best fitting model for the three groups was logarithmic, but the R^2 value increased with age (youngest: R^2 log = 59%, $p < .01$, vs. R^2 lin = 46%, $p < .05$, $t[9] = -1.2$, $p > .05$; intermediate: R^2 log = 85%, $p < .001$, vs. R^2 lin = 57%, $p < .05$, $t[9] = -3.15$, $p < .05$; and oldest: R^2 log = 94%, $p < .001$, vs. R^2 lin = 70%, $p < .01$, $t[9] = -3.62$, $p < .01$).

Individual analysis. Regression analyses were performed on the data of individual children. The best fitting model between linear and logarithmic was attributed to each child, whenever significant (e.g., the child was attributed a logarithmic representation for a given interval if the highest R^2 was logarithmic). If both failed to reach significance, the child was classified as not having a representation for the interval considered.[2] For each interval, children were therefore classified as having a linear representation, a logarithmic representation, or no representation (see Table 1).

Spearman rank order correlations were calculated between group (ordered by age: 1 = youngest, 2 = intermediate, and 3 = oldest) and type of representation (ordered by developmental phase: 1 = no representation, 2 = logarithmic, and 3 = linear).[3] For the small interval the estimation tended to become linear with age ($r_s = .43$, $p < .005$, one-tailed test), whereas for the bigger interval the correlation reflected an increase of logarithmic representation ($r_s = .31$, $p < .05$, one-tailed test). Overall, children show more accurate positioning of numbers with increasing age. When the numerical context is difficult, or unfamiliar, they rely on an intuitive, logarithmic representation, whereas when the numerical context is familiar, they use a linear representation. However, irrespective of the best fitting model attributed to each child, we observed that linearity, indexed by the linear R^2, increased with age group on both intervals (ANOVAs: 1–10 interval, $F[2, 43] = 4.47$, $p < .05$, $\eta^2 = .04$; 0–100 interval, $F[2, 43] = 4.83$, $p < .05$, $\eta^2 = .07$). The linear R^2 for the two intervals was also correlated with age in months (1–10 interval: $r = .35$, $p < .05$; 0–100 interval: $r = .45$, $p < .01$, both one-tailed).

Table 1
Type of Representation as a Function of Group and Task in Experiment 1

Task	Type of representation		
	None	Logarithmic	Linear
1–10 interval			
Youngest (n = 11)	46	36	18
Intermediate (n = 16)	31	19	50
Oldest (n = 19)	5	16	79
0–100 interval			
Youngest (n = 11)	64	27	9
Intermediate (n = 16)	56	38	6
Oldest (n = 19)	21	68	11

Note. Cell values represent percentages (per row) of children.

This pattern is also supported by the analyses conducted on both intervals according to the type of representation (see Table 2). The ability to position numbers on one interval was significantly correlated with the ability to position numbers on the other interval when age group was partialled out ($r_s = .27$, $p < .005$, one-tailed). In other words, children with a more precise representation on the 1–10 interval also had a better representation on the 0–100 interval. This result supports the developmental trend of a logarithmic representation prior to a linear one and an improvement first on a familiar smaller context and then on a less familiar or harder context.

Naming scores, from youngest to oldest group, were on average 4 (SD = 3.6), 6.3 (SD = 3.6), and 9.1 (SD = 1.2), respectively. Correlations with type of representation on the two intervals were significant when age group was partialled out (interval 0–10: $r_s = .30$, $p < .002$, one-tailed; interval 0–100: $r_s = .36$, $p < .001$, one-tailed). These results suggest that a better knowledge of Arabic numerals goes together with the use of more precise numerical representations—linear for the smaller interval and predominantly logarithmic for the larger interval—in the number-to-position task.

Experiment 2

Method

Participants. A new population of 373 children (200 girls) recruited in several kindergarten schools from northeastern Italy took part in the study. They were all from the Italian ethnic group and mostly of middle socioeconomic status. They were divided into three groups as in Experiment 1. The youngest group (n = 74) had a mean age of 48 months (range: 43–53), the middle group (n = 128) had a mean age of 59 months (range: 54–64), and the oldest group (n = 168) had a mean age of 70 months (range: 65–75).

Procedure. The procedure was identical to that of Experiment 1, but the 1–100 interval was replaced by a novel 1–20 interval (Items 2, 4, 6, 7, 13, 15, 16, and 18; Item 10 as practice). Children were first asked to count on their fingers up to 10 and to order by magnitude Arabic digits and dots ranging from 1 to 5 (presented on cards of ≈ 4 cm^2); the three tasks were scored with 1 point for each correct item. Children had no time pressure and could stop at any time.

Results

The data were analyzed as in Experiment 1. The ANOVA showed a significant effect of age for both intervals (1–10: $F[2, 367] = 4.15$, $p < .05$, $\eta^2 = .02$; 1–20: $F[2, 357] = 37$, $p < .001$,

[2] Among those children classified as not having a linear or a logarithmic representation, some used evident nonnumerical strategies such as alternating between left and right marks on the lines.

[3] The small observed frequencies in some cells prevented us from running a chi-square analysis in Experiment 1.

Table 2
Relation Between Types of Representation Across Tasks in Experiment 1

1–10 interval	0–100 interval: None	Logarithmic	Linear
None	19.6	2.2	2.2
Logarithmic	8.7	13.0	0.0
Linear	15.2	32.6	6.5

Note. Cell values represent percentages of children (n = 46) who adopt a given combination of representations across tasks.

η^2 = .17; respectively, 3 and 13 children were excluded for not completing enough items), indicating that the *PE*s decreased with age (1–10 interval: 24%, 22%, 20%; 1–20 interval: 28%, 20%, 15%, for the three age groups, respectively). The youngest group was significantly less accurate than the oldest (p < .01) on the 1–10 interval, and all three age groups were significantly different on the 1–20 interval (ps < .001).

Group analysis. On group medians (see Figure 2), the pattern of estimates for the 1–10 interval was fitted equally well by a logarithmic or linear function in the youngest group (R^2 log = 98%, p < .001, R^2 lin = 97%, p < .001), $t(6)$ = 1.03, p = .34. For the two other groups, however, the linear model yielded a better fit (intermediate group: R^2 log = 92%, p < .001, R^2 lin = 98%, p < .001, $t[6]$ = –3.17, p < .05; oldest group: R^2 log = 89%, p < .001, R^2 lin = 97%, p < .001, $t[6]$ = –5.5, p < .005). This replicates the findings of Experiment 1. For the 1–20 interval, the estimates of the youngest group were best fitted by a logarithmic function (R^2 log = 94%, p < .001, R^2 lin = 77%, p < .005, $t[7]$ = 2.79, p < .05). The fits for the older groups did not significantly differ between logarithmic and linear models (intermediate group: R^2 log = 97%, p < .001, R^2 lin = 98%, p < .001, $t[7]$ = –0.95, p = .38; oldest group: R^2 log = 96%, p < .001, R^2 lin = 99%, p < .001, $t[7]$ = –1.86, p = .10). These results are in line with previous findings showing a mandatory logarithmic phase prior to linearity.

Individual analysis. As in Experiment 1, individual children were classified as having a linear representation, a logarithmic representation, or no representation for each interval (see Tables 3 and 4). The type of representation varied with the children's age for both intervals, $\chi^2(4)$ = 48, p < .001, and $\chi^2(4)$ = 58, respectively, ps < .001. The correlations between type of representation on each interval and age group were significant, indicating an improvement of type of positioning with age (interval 1–10: r_s = .37, p < .001, one-tailed test; interval 1–20: r_s = .37, p < .001, one-tailed test). The correlation between representations on the two intervals, with age group partialled out, was also significant (r_s = .51, p < .001, one-tailed test).

One way ANOVAs on the linear R^2 for both intervals showed that linearity significantly improved with age group (1–10 interval: $F[2, 367]$ = 24, p < .001, η^2 = .03; 1–20 interval: $F[2, 367]$ = 41,

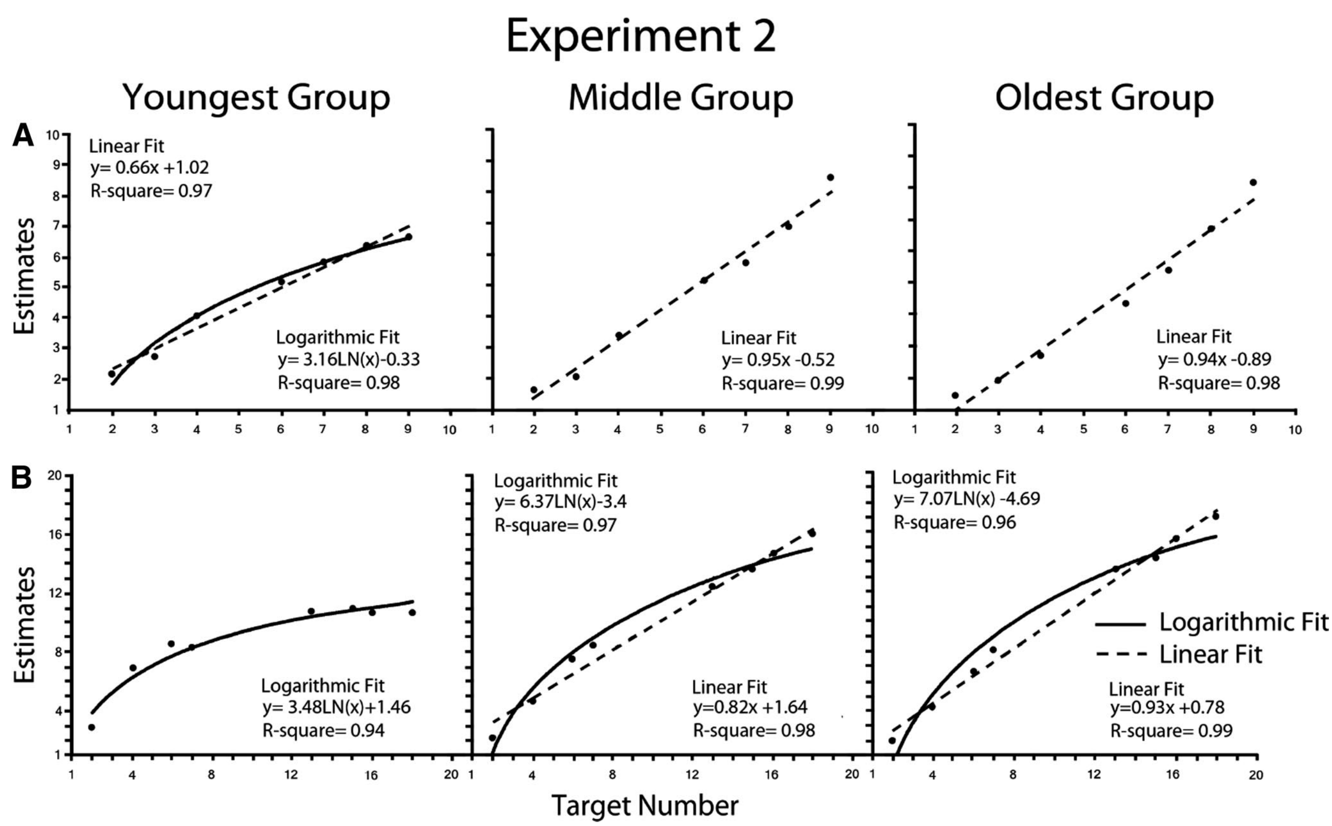

Figure 2. Children estimates and best fitting models in Experiment 2 as a function of age group for 1–10 (A) and 1–20 (B) number lines.

Table 3
Type of Representation as a Function of Group and Task in Experiment 2

Task	Type of representation		
	None	Logarithmic	Linear
1–10 interval			
Youngest (n = 75)	52	21	27
Intermediate (n = 130)	38	17	45
Oldest (n = 168)	15	14	71
1–20 interval			
Youngest (n = 75)	66	15	19
Intermediate (n = 130)	38	28	34
Oldest (n = 168)	16	31	53

Note. Cell values represent percentages (per row) of children.

$p < .001$, $\eta^2 = .06$). The correlation between linear R^2 and age in months was significant for both intervals (1–10 interval: $r = .33$, $p < .001$; 1–20 interval: $r = .39$, $p < .001$, both one-tailed).

Mean counting on finger scores were 7.3 (SD = 2.8), 8.98 (SD = 2.1), and 9.45 (SD = 1.6) from youngest to oldest, respectively. Ordering scores (from youngest to oldest) were on average 2.3 (SD = 2), 3.8 (SD = 1.9), and 4.7 (SD = 1) for Arabic digits and were 2.2 (SD = 1.9), 3.8 (SD = 1.8), and 4.5 (SD = 1.2) for dots. Partial correlations were significant between type of representation and performance in the ordering tasks (1–10: $r_s = .30$, $p < .001$, for dots, and $r_s = .31$, $p < .001$, for Arabic numbers; 1–20: $r_s = .35$, $p < .001$, for dots, and $r_s = .24$, $p < .001$, for Arabic numbers), but not in the counting sequence task (1–10: r_s = .02, $p = .6$; 1–20: $r_s = -.06$, $p = .28$). The latter finding may not be attributed to a ceiling effect since a one-way ANOVA on counting scores showed a statistical difference between groups, $F(2, 364) = 28$, $p < .001$.

Conclusions

The present study shows that an understanding of how numbers map onto space develops long before formal education begins. In Experiment 1, preschoolers deployed a logarithmic positioning when confronted with the 0–100 number range, and the youngest children showed a trend toward a logarithmic positioning even for the 1–10 interval. In contrast, older children deployed a linear strategy when confronted with the more familiar range of small numbers (1–10 interval), and the oldest group approximated very closely the ideal positioning. In Experiment 2, on the 1–20 interval the youngest children deployed a logarithmic positioning, whereas the two other groups started to show some abilities to position numbers linearly. Thus, the use of a logarithmic strategy before a linear one seems mandatory even in the small number range.

The dissociation between smaller and larger intervals is consistent with the results of Siegler and colleagues (Siegler & Booth, 2004; Siegler & Opfer, 2003) and reveals the coexistence of multiple spatial representations for numbers. The youngest age groups in their studies relied on a linear positioning for the 0–100 number line (our larger interval), whereas on the 0–1,000 number line only the oldest group of children (Grade 6) was able to position numbers linearly like adults. Note that Grade 4 children in Siegler and Opfer's (2003) study used a logarithmic strategy for the 0–1,000 interval, although at this age children are already able to count as far as 1,000 and are also familiar with simple divisions. Therefore, knowing the numerical sequence does not seem to be the only prerequisite to apply a linear strategy (indeed counting scores did not correlate with type of representation in Experiment 2). It rather implies a representational change either by a shift from a logarithmic to a linear representation or by creating a complementary representation of exact numbers (for models of exact number representation, see Verguts, Fias, & Stevens, 2005; Zorzi & Butterworth, 1999).

In Whyte and Bull's (2008) intervention study, only children who played with a linear numerical board game (compared to nonlinear numerical and linear color board games) became more accurate and linear at posttest in positioning numbers on a 0–10 number line. Interestingly, Geary and colleagues (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Geary, Hoard, Nugent, & Byrd-Craven, 2008) found that children with mathematical learning disability were less accurate in numerical estimation (on a 0–100 number line) and more reliant on logarithmic representation than typically achieving peers. Moreover, Geary and colleagues observed trial-by-trial variation in the use of logarithmic or linear strategy. Note that the best fitting equations from our Experiment 2 could serve as reference measure of group performance to investigate trial-by-trial variation in future studies (see Geary et al., 2007).

Logarithmic coding of numbers (Dehaene et al., 2003) is a hallmark of the approximate number system subserving the nonsymbolic representation of numerosities (Feigenson et al., 2004; see Dehaene & Changeux, 1993, for a computational model). The finding that the logarithmic fit over the three groups and the accuracies of estimates increased with age suggests a developmental pattern even for the logarithmic representation. Indeed, increasing precision of the logarithmic representation is consistent with the finding that the ability to discriminate the numerosity of two sets increases with age (Halberda & Feigenson, 2008; Lipton & Spelke, 2003).

Lipton and Spelke (2005) have shown that preschool children map the number words within their counting range onto nonsymbolic representations of numerosity but that they show no such mapping for number words beyond that range. Moreover, according to the "enriched parallel individuation system" hypothesis of Le Corre and Carey (2007), children need to learn the counting principles before being able to map exact numerical symbols beyond four to analog magnitudes. Indeed, the youngest children in our study showed a poor and inconsistent performance in the

Table 4
Relation Between Types of Representation Across Tasks in Experiment 2

1–10 interval	1–20 interval		
	None	Logarithmic	Linear
None	24.1	2.7	3.5
Logarithmic	4	7.8	4.8
Linear	5.9	16.4	30.8

Note. Cell values represent percentages of children (n = 373) who adopt a given combination of representations across tasks.

0–100 interval, whereas they mastered the estimation task in the 1–10 interval and some started to master the 1–20 interval. This result is in line with Le Corre and Carey's results, since their children were able to map numbers larger than four only around age 4.5. Our children in the youngest group had a mean age of 4, and over a half of them (54% and 60% for Experiments 1 and 2, respectively) where able to estimate numbers following either the logarithmic or linear positioning on the 1–10 interval.

Finally, the precision of numerical estimation across all children in our study was correlated with their ability to name single-digit Arabic numbers as well as ordering numbers from 1 to 5. This finding highlights the role of mastering numerical meaning and symbols—and hence of formal instruction—in structuring the child's understanding of numbers, although the exact path that leads from a logarithmic to a linear representation remains to be understood (for theoretical suggestions, see Dehaene, 2007; Verguts & Fias, 2004).

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