exactly what researchers found (Dehaene & Akhavein 1995; Ganor-Stern & Tzelgov 2008), and they reasonably concluded that numerical symbols automatically activated their semantic meaning.

Researchers should remember that, despite its name, the *numerical distance effect* is correlational. That is, numerical distance is one of several features that are correlated with the order of the numbers on the number line. Therefore, although numerical distance is correlated with participants' RTs, it may not be the controlling factor of participants' RTs. As with all correlational data, there may be a third variable that (1) is correlated with numerical distance and (2) is the true controlling factor of participants' responses. The first step in determining whether a third variable exists is to test alternative hypotheses. Unfortunately, in numerical cognition research, researchers rarely (if ever) consider plausible alternatives to the numerical distance hypothesis. Recently, I did just that when I tested whether numerical symbols automatically activate their representation.

I ran the numerical same/different task with two simple changes: (1) I did not dichotomize the stimuli, and (2) I tested an alternative hypothesis. By not dichotomizing the levels of the independent variable, I reduced the number of plausible alternative hypotheses that could explain the data. The plausible alternative I tested was simple: RT increased as a function of the *physical similarity* between the two numbers to be distinguished. See Cohen (2009) for the operational definition of the physical similarity function. The data were clear: Physical similarity was the controlling factor in participants' RTs, not numerical distance. Because the semantic meaning did not interfere with participants' response, the data demonstrated that integers do not automatically activate their semantic meaning. *Because the numerical distance function correlates highly with the physical similarity function* ($r = .62$), *researchers may easily confuse the effects of physical similarity for those of numerical distance if they do not actively test both*.

CK&W benefit the numerical cognition community by reviewing the literature addressing the abstract nature of numerical representations. The authors remind us not to confuse correlations for causal mechanisms. I echo that sentiment and remind readers that the numerical distance effect is, at its essence, simply a correlation. By challenging the tautologies of the field with plausible alternative hypotheses, researchers like Cohen Kadosh & Walsh keep the numerical cognition field moving forward.

ACKNOWLEDGMENT
This work was supported by NIH grant RO1HD047796.

The case for a notation-independent representation of number

doi:10.1017/S0140525X09990033

Stanislas Dehaene
Inserm-CEA Cognitive Neuroimaging Unit, NeuroSpin Center, CEA/SAC/DSV/I2BM, Bât 145, Point Courrier 156, F-91191 Gif/Yvette, France.
Stanislas.Dehaene@cea.fr
http://www.unicog.org

Abstract: Cohen Kadosh & Walsh (CK&W) neglect the solid empirical evidence for a convergence of notation-specific representations onto a shared representation of numerical magnitude. Subliminal priming reveals cross-notation and cross-modality effects, contrary to CK&W's prediction that automatic activation is modality and notation-specific. Notation effects may, however, emerge in the precision, speed, automaticity, and means by which the central magnitude representation is accessed.

Cohen Kadosh & Walsh (CK&W) revive a twenty-year-old proposal in numerical cognition, according to which our capacity for abstract mathematical thought results from a complex interaction of multiple concrete notation-specific codes (Campbell & Clark 1988). Their conclusions, unfortunately, result from discarding of solid empirical evidence supporting a convergence of notation-specific representations onto a shared representation of numerical magnitude, in favor of weak evidence for notation effects. Thus, their article consists in a catalogue of findings in which any difference or interaction involving number notation is taken as strong support for the notation-specific view. In this commentary, I first return to the evidence for notation-independent representations, then discuss interesting reasons why notation occasionally influences brain and behavior.

Evidence for notation-independent number processing. CK&W cite, but do not seem to draw conclusions from, the many functional magnetic resonance imaging (fMRI) studies that have observed shared fMRI activations across different notations for numbers. Most important, are studies showing cross-notation fMRI adaptation, because they cannot be dismissed as just showing activation overlap, or as resulting from artifacts of response selection (Jacob & Nieder 2009; Naccache & Dehaene 2001a; Piazza et al. 2007). As an example, extending earlier work by Piazza et al. (2007), Jacob and Nieder (2009) recently demonstrated that after adaptation to a fraction expressed with Arabic numerals (e.g., 1/2), anterior intraparietal sulcus (IPS) shows a distance-dependent transfer of adaptation to number words (e.g., half). As a second example, Figure 1 replots the data in Naccache and Dehaene (2001a), showing that IPS activation is reduced whenever the same numerical quantity is repeated, regardless of whether notation is changed (Arabic digits vs. number words). Note that in this experiment, the first presentation of the number was subliminal, and yet the results showed clear cross-notation convergence. In general, it is surprising that CK&W do not cite the extensive behavioral literature on subliminal priming demonstrating clear notation-independent effects (e.g., Dehaene et al. 1998b; Reynvoet & Brysbaert 2004; Reynvoet et al. 2002). Even cross-modal subliminal priming was recently demonstrated, from a subliminal Arabic numeral to a conscious spoken numeral (Kouider & Dehaene, in press). These effects argue directly against CK&W's proposal that when numerical representations are probed implicitly, convergence across notations does not occur.

Equally impressive are electrophysiological studies showing that some single neurons, particularly in prefrontal cortex, respond identically to symbolic and various non-symbolic displays of number, with the same exact tuning curve across the interval of numbers tested (Diester & Nieder 2007; Nieder et al. 2006). Astonishingly, CK&W dismiss these beautiful data with the claim that the animals are using a "similar response strategy," for example, for digit 1 and dot 1 compared to other numbers – not acknowledging the fact that their interpretation would require as many putative strategies as there are numbers!

Returning to fMRI, another method that is likely to play a strong role in the coming years is multivariate decoding, which probes fMRI activation for the presence of decodable patterns and their generalization to novel experimental conditions. Using this technique, with Evelyn Eger (Eger et al., submitted), we recently demonstrated that human IPS signals can be used to decode which number a participant is temporarily holding in mind. When trained with Arabic numerals, the IPS decoder generalizes to dot patterns, indicating the presence of an underlying notation-independent neuronal population. Likewise, with André Knops (Knops et al. 2009), we found that a classifier trained with posterior IPS activation during left versus right saccades could spontaneously generalize to a classification of subtraction versus addition trials, whether these calculations were performed with non-symbolic sets of dots or with symbolic Arabic numerals.

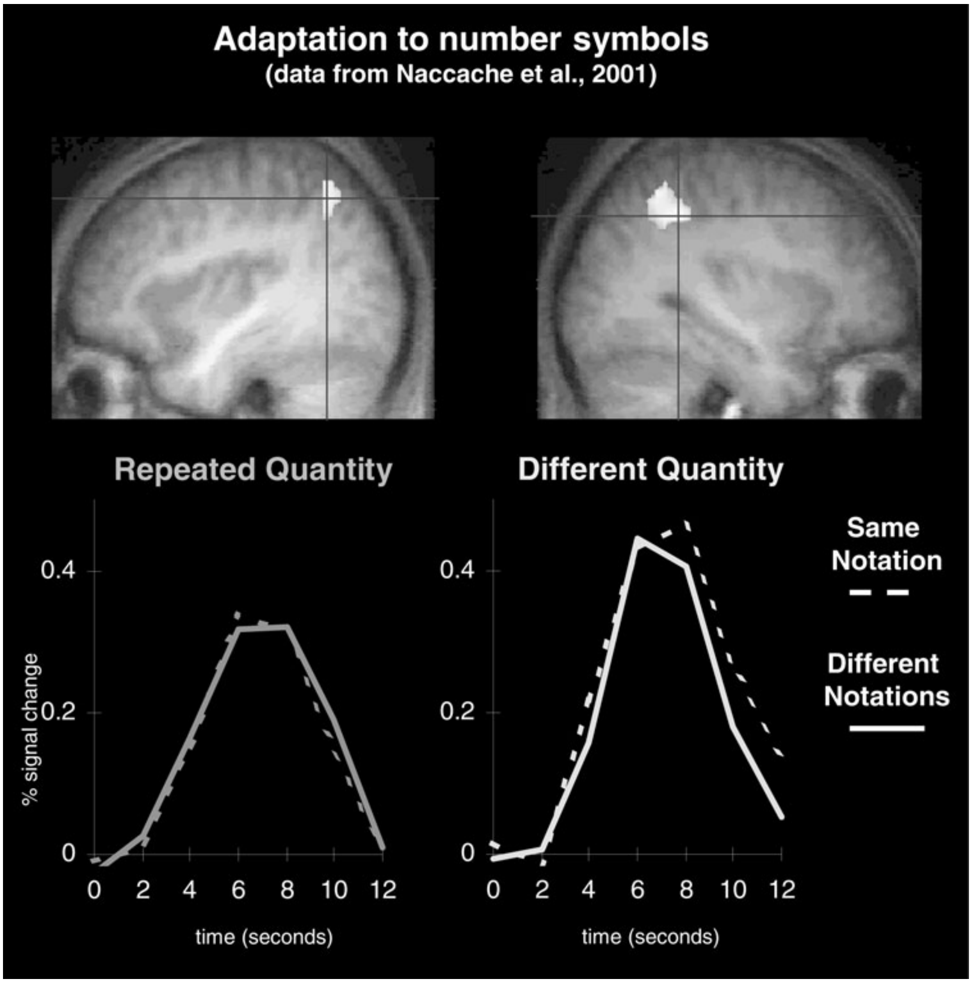

Figure 1 (Dehaene). Evidence for cross-notation subliminal priming and access to a shared representation of Arabic and verbal numerals. Naccache and Dehaene (2001a) presented a subliminal prime prior to each target number in a number comparison task. Using whole-brain fMRI, repetition suppression was observed only in bilateral intraparietal cortex: Trials in which the primes and targets corresponded to the same quantity yielded reduced activation compared to trials in which the quantities differed. Response times were also accelerated by subliminal repetition. In both fMRI and behavior, it made no difference whether the prime and the targets appeared in the same or different notations. Reynvoet et al. (2002) later expanded this work by showing that the amount of priming in response times varies continuously as a function of the numerical distance between the prime and target.

The concept of "abstract representation" is never defined by CK&W, and here the Knops et al. (in press) findings point to an important theoretical point: A representation may be shared across numerical notations, and yet rely on a spatial, non "abstract" format of representation. Knops et al. show that the putative human homolog of monkey area LIP, a retinotopic region involved in attention and eye movement, is partially co-opted for symbolic *and* non-symbolic arithmetic on the number line – a clear instance of "cortical recycling" of a sensorimotor area for a more abstract mathematical use (Dehaene & Cohen 2007).

Interesting reasons why number notation occasionally influences brain and behavior. If numbers presented in various notations contact unified representations of magnitude and space, then why are significant notation effects occasionally observed? Several explanations can be proposed, none of them requiring a hasty dismissal of notation-independent representations.

1. *Numerical precision.* Neural network simulations suggest that the introduction of symbolic representation can lead to a refined precision of the tuning curves for number (Verguts & Fias 2004). Importantly, according to this view, the *same* neurons remain responsive to both symbolic and non-symbolic presentations – only their accuracy changes. Mathematical developments of this theory (Dehaene 2007) suggest that it has the potential to explain many of the known effects of education to symbols on mental arithmetic, including the linearization of number-space mappings (Dehaene et al. 2008; Siegler & Opfer 2003), the improved accuracy with which Arabic numerals can be compared or combined into calculations, and the spontaneous competence of young children when symbols are first introduced (Gilmore et al. 2007) – an effect hard to explain without assuming cross-notation convergence.

2. *Speed and automaticity*. There is no reason to expect all number notations to be equally fast and automatic in accessing the shared magnitude representation. On the contrary, identification is slower for number words than for Arabic numerals. In number comparison, interactions between the distance effect and the verbal versus Arabic notation of the targets can be attributed to a word-length effect unique to written words (Dehaene 1996). Education and over-training also play a role – as children get older, increasingly automatic effects of numerical magnitude are seen, particularly with Arabic numerals (Girelli et al. 2000). These notation effects occur at a perceptual or transcoding level and are largely irrelevant to the existence of a shared central representation for number.

3. *Neural machinery for transcoding.* In the course of converting from a notation-specific neural code to a numerical magnitude code, it is likely that the brain requires special neural

machinery, possibly including neurons that are attuned to both quantity *and* notation. Indeed, in parietal cortex, neurons with a joint sensitivity to multiple parameters (e.g., eye position *and* retinal location) are frequently recorded and are thought to play a compulsory role in transcoding to and from pure representations of each parameter (Pouget et al. 2002). Similarly, neurons simultaneously tuned to both numerical magnitude and notation (e.g., simultaneous dots vs. serial flashes [Nieder et al. 2006]; or dots versus Arabic symbols [Diester & Nieder 2007]) may play a key role in accessing a common magnitude representation via different routes. Identifying the numerosity of a set of dots may require special neural machinery for summing size-invariant representations of object locations (Dehaene & Changeux 1993), potentially explaining both the existence of neurons monotonically tuned to non-symbolic numerosity in area LIP (Roitman et al. 2007) and the specific form of behavioral priming observed with dots compared to Arabic numerals (Roggeman et al. 2007).

Conclusion. Considerable evidence points to a notation-independent representation of number in the monkey and human IPS. It would, however, be wrong to think of this representation as a "module." First, it does not involve a dedicated area, but only a fraction of IPS neurons, highly distributed in the IPS, and intermingled with other representations of extent, time, location, and other continuous parameters (Pinel et al. 2004; Tudusciuc & Nieder, 2007). Second, it is not "encapsulated," but communicates broadly with other areas, thus allowing humans to attach arbitrary spoken and written symbols to it. Because it acquires its "abstract" character, in large part, through education to number symbols, it is not surprising that notation effects are occasionally seen. Such effects should not divert from the incontrovertible fact that arithmetic, like much of mathematics, consists in the manipulation of internal conceptual representations that abstract away from the specific format of input.

Concrete magnitudes: From numbers to time

doi:10.1017/S0140525X09990045

Christine M. Falter,[a] Valdas Noreika,[b] Julian Kiverstein,[c] and Bruno Mölder[d]

[a]*Department of Psychiatry, University of Oxford, Warneford Hospital, Oxford OX3 7JX, United Kingdom;* [b]*Centre for Cognitive Neuroscience, Department of Psychology, University of Turku, Turku 20014, Finland;* [c]*Department of Philosophy, School of Philosophy, Psychology and Language Sciences, University of Edinburgh, Edinburgh EH8 9AD, Scotland, United Kingdom;* [d]*Department of Philosophy, Institute of Philosophy and Semiotics, University of Tartu, Tartu 51003, Estonia.*

Christine.Falter@psych.ox.ac.uk
http://www.alice-dsl.net/subjective.time/index.html
valnor@utu.fi **j.kiverstein@ed.ac.uk**
bruno.moelder@ut.ee **http://moelder.wordpress.com**

Abstract: Cohen Kadosh & Walsh (CK&W) present convincing evidence indicating the existence of notation-specific numerical representations in parietal cortex. We suggest that the same conclusions can be drawn for a particular type of numerical representation: the representation of time. Notation-dependent representations need not be limited to number but may also be extended to other magnitude-related contents processed in parietal cortex (Walsh 2003).

Number (Piazza et al. 2004) as well as time (Rao et al. 2001) have been found to be represented in the intraparietal sulci (IPS). The mode of their representation – abstract or not abstract – is put into question by evidence uncovered by Cohen Kadosh & Walsh (CK&W). The authors define *abstract* representation as the insensitivity of neuronal populations coding numerical quantity to the notation in which the numerical quantity was presented. CK&W elegantly show in their careful scrutiny of existing studies that this property cannot be attributed to all representations of number. We suggest that their conclusion may be extended to representations of time.

We will refer to representations that are notation-specific as *concrete* representations. We will argue that primary neuronal representations of time (which we speculate may also be found in IPS) could be concrete. CK&W take as their measure of concreteness, the interaction of variations in behavioural data or neuronal activation with different numerical notations. We will show that comparable interactions are found in behavioural performance relating to notation- and modality-specific representations of time. First, we describe some studies that suggest that the representations of number we use to tell the time from clocks are concrete. Second, we will discuss studies that suggest that duration perception may also be concrete.

In a study of clock reading behaviour in children, Friedman and Laycock (1989) found an interaction effect. They asked 6- to 11-year-old children to detect the time displayed in pictures of analog and digital clocks. The success at reading the time from analog displays systematically varied with presented time, but this was not the case for digital displays. The independent variation in performance supports a view of representations of clock time as concrete.

Clock time also seems to be represented concretely in adults. Goolkasian and Park (1980) also found an interaction of notations with presented time. They asked subjects to compare clock times presented as words, with a second clock time presented either in analog or digital notation. There was an effect of the angular distance between the two times if the second stimulus was a clock face as compared to a digital display. This corroborates the finding by Friedman and Laycock (1989) in children and shows that concrete representations of clock time do not disappear during development.

Support for concrete representations of clock time also comes from repetition priming tasks. In a study by Meeuwissen et al. (2004), participants had to name the time from analog clocks, followed by naming the time from a digital clock. Although there was no obvious repetition priming effect with respect to the hour, there was a facilitation present for naming the minutes. If subjects used an abstract representation to name the hour, we would expect to find a priming effect. The absence of such a priming effect suggests that the representations used in naming the hour are concrete. The authors interpreted this finding as evidence that we use different strategies to determine the hour, depending on whether we are using an analog clock or a digital display. Since determining the hour is the first stage in telling the time, we conclude that at this first stage, concrete representations of clock time are employed. Meeuwissen and colleagues showed that once the hour is known, determining the distance in minutes from the hour (e.g., 5 minutes before or after an hour) relies on the same operation for both display formats. This two-stage account of the processing of clock time is consistent with the model CK&W advance concerning the possibility of transformation of primary concrete numerical representations in parietal areas into a common format depending on task demands.

Representations of clock time seem to exert notation-dependent effects on behaviour. We propose that comparable interaction effects can also be found for duration perception, suggesting that the brain may represent temporal intervals concretely. Smith et al. (2007) employed a duration bisection procedure to examine interval timing of two different temporal ranges, 100–500 msec and 1–5 sec, in patients with Parkinson's disease and healthy controls. The authors found a significant interaction between the temporal ranges and modality (visual vs. auditory). Both patients and controls overestimated visual short durations and underestimated visual long durations, but this effect was not observed in the case of auditory durations. This pattern in performance suggests that duration perception is modality-specific, hence concrete.