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Dissociable Mechanisms of Subitizing and Counting: Neuropsychological Evidence From Simultanagnosic Patients

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Do people have to count to determine visual numerosity, or is there a fast "subitizing" procedure dedicated to small sets of 1–3 items? Numerosity naming time and errors were measured in 5 simultanagnosic patients who suffered from severe difficulties in serial counting. Although these patients made close to 100% errors in quantifying sets comprising more than 3 items, they were excellent at quantifying sets of 1, 2, and sometimes 3 items. Their performances in visual search tasks suggested that they suffered from a deficit of serial visual exploration, due to a fundamental inability to use spatial tags to keep track of previously explored locations. The present data suggest that the patients' preserved subitizing abilities were based not on serial processing but rather on a parallel algorithm dedicated to small numerosities. Several ways in which this parallel subitizing algorithm might function are discussed.

After almost a century of psychological research, the existence of a dedicated mental process for quantifying[1] small sets of items remains highly controversial. Kaufman, Lord, Reese, and Volkmann (1949) coined the term *subitizing* to refer to the unknown process by which subjects rapidly and accurately report the numerosity of small sets. However, the extreme speed of identification of small numerosities was recognized before the beginning of this century (e.g., Bourdon, 1908; Cattell, 1886; Jevons, 1871; Warren, 1897). Initial research indicated that subjects were fast and accurate at quantifying sets of up to 6 or 7 items (Jensen, Reese, & Reese, 1950; Taves, 1941). With the advent of more accurate chronometric techniques, however, it was recognized that performance in the 4–7 range was probably imputable to serial counting on the basis of an iconic memory of the display (Averbach, 1963; Chi & Klahr, 1975; Klahr, 1973; Mandler & Shebo, 1982; see also Warren, 1897). What remained unclear was whether sets of 1, 2, 3, and perhaps 4 items were also quantified by counting, or whether they were processed by a dedicated subitizing procedure.

Subitizing Controversy

Experimenters have typically measured the speed and accuracy of normal adults at naming the number of items in a briefly presented array (e.g., 200 ms). The typical findings appear in Figure 1. First, accuracy is close to perfect over the 1–3 range and starts to drop regularly with 4 or more items. Second, naming time increases slowly and nonlinearly from 1 to 3 items, and then starts to increasing sharply and linearly, by about 200 ms/item, for sets of 4 or more items.

Subscribers to the hypothesis that subitizing is radically different from counting have often caricatured these results. In particular, it is not the case that quantification time is constant over the subitizing range of 1 to 3 items, or even that it increases linearly with a rate of about 40 ms/item from 1 to 3 items, and then jumps to a rate of about 200–400 ms/item (e.g., Akin & Chase, 1978; Chi & Klahr, 1975; Oyama, Kikuchi, & Ichihara, 1981; see also Sagi & Julesz, 1985). Rather, numerosity naming time generally increases by at most 20 ms from 1 to 2 items, by about 50 ms from 2 to 3 items, and by 100–200 ms from 3 to 4 items (Jensen et al., 1950; Mandler & Shebo, 1982; Saltzman & Garner, 1948). Because the time to name a single digit is roughly constant (e.g., Mandler & Shebo, 1982), this increase cannot be attributed to a confounding variable such as word frequency. Rather, it is a perceptual increase in difficulty that can even be demonstrated in tasks that do not require overt verbal production. For instance, for a fixed duration of presentation, it is easier to discriminate 1 versus 2 than 2 versus 3, and 2 versus 3 than 3 versus 4 (Folk, Egeth, & Kwak, 1988).

Some researchers claim to have identified a sharp discontinuity in quantification performance and have equated it with the end of the subitizing range and the beginning of counting. Akin and Chase (1978), Chi and Klahr (1975), Oyama et al. (1981), Simons and Langheinrich (1982), and

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[1] Following Klahr (1973), we use the neutral terms *quantify* and *quantification* for any process that may determine the numerosity of a visual display. The term *counting* is reserved to the specific process of serial one-to-one correspondence between a list of objects and a list of numerons, as defined by Gelman and Gallistel's (1978) counting principles.

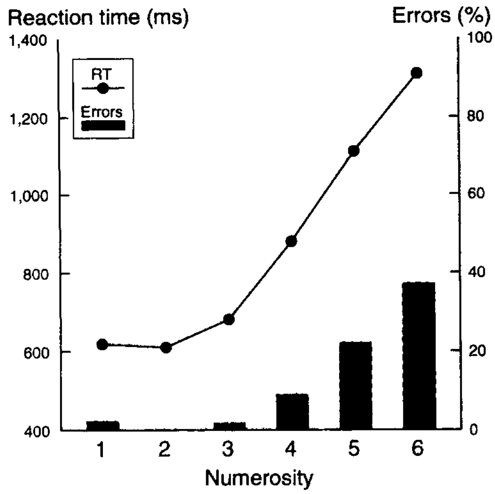

Figure 1. Reaction time (RT) and accuracy for naming the numerosity of a random set of one to six letters presented visually for 200 ms. Redrawn from "Subitizing: An analysis of its component processes" by G. Mandler and B. J. Shebo, 1982, *Journal of Experimental Psychology: General, 111,* p. 14. Copyright 1982 by the American Psychological Association. Adapted with permission of the author.

Trick and Pylyshyn (1993, 1994) described discontinuities between 4 and 5 items in the slope of response times. Atkinson, Campbell, and Francis (1976; Atkinson, Francis, & Campbell, 1976) found a sudden onset of errors and slow responses between 4 and 5 items. Mandler and Shebo (1982) also found a sudden increase, between 3 and 4, in the time to quantify random displays relative to displays with a reproducible "canonical" arrangement of objects (see also Warren, 1897).

As may be inferred from this list, the subitizing limit varied from one study to the other, although it was always situated around 4. In addition, the purported discontinuities were rarely submitted to statistical testing. Several researchers have preferred to emphasize the continuous decrease of performance as a function of display size (e.g., Averbach, 1963; Hunter & Sigler, 1940; Saltzman & Garner, 1948; van Oeffelen & Vos, 1982). At least two recent articles, using modern statistical techniques, have seriously questioned the existence of any discontinuity in enumeration times in the 1–8 range (Balakrishnan & Ashby, 1991, 1992).[2]

Subitizing as Fast Counting

According to Gallistel and Gelman (1991, 1992), subitizing is nothing but fast counting. Reviewing the literature on numerosity perception in animals, they suggested the existence of a fast nonverbal counting procedure whose accuracy decreases with numerosity. They proposed that nonverbal counting persists in human infants and adults and accounts for the fast enumeration of small displays. The linear increase in naming time for numerosities above 4 would be due to subjects shifting to a slower mode of verbal counting.

The fast counting model readily explains that RTs increase with numerosity even within the putative subitizing range. The nonlinearity of the RT curve seems more problematical, but there are in fact a number of ways to accommodate it within a counting model. First, different subjects may switch to the slow mode of verbal counting for different levels of numerosity. This would smear the boundary between slow and fast counting and would give the impression of a continuous increase in slope as a function of numerosity. Second, even if there was only a single process of verbal counting, its rate might not remain constant with numerosity. Counting implies keeping track of previously counted items and searching for remaining ones. Both of these processes might become slower for larger numerosities. Memory load, in particular, might be very light for numerosities as small as 1, 2, or 3, thereby explaining that they can be rapidly enumerated. In summary, the entire enumeration time curve might be accounted by a single process of counting.

Recently, Trick and Pylyshyn (1993, 1994) presented new evidence that subitizing relies mostly on a parallel preattentive process rather than a serial counting process. They showed that the numerosity naming time curve becomes linear, and that subitizing therefore seems to disappear, in conditions that prevent parallel processing of the target set (quantifying *O*s among distracting *Q*s, conjunctions of visual features, or sets of concentric rectangles). They also found a significant effect of attentional cuing on naming times outside the subitizing range, but little or no effect for numerosities of up to 3 or 4 items.

Although these data are impressive, it is not clear that they could not be accounted for in a counting model. Serial processes such as counting must be guided by earlier preattentive processing (Wolfe, Cave, & Franzel, 1989). For attention to move to the next counted item, other processes must have separated this item from the background and signaled it as a potential target (Trick & Pylyshyn, 1994). Manipulations that make the quantified set less obvious, such as surrounding a set of *O*s with distracting *Q*s, may interfere with such preattentive labelling processes and therefore reduce the counting speed for small numerosities. They would be expected to have less effect on larger numerosities because this part of the numerosity naming time curve is already dominated by the slower strategic processes of verbal counting. On this account, subitizing would still be nothing but preattentively guided serial counting. In fact, Trick and Pylyshyn (1994) stated that they do not argue that subitizing is a parallel process. Yet other models, to be reviewed in the General Discussion section, view parallel processing or "immediate apprehension" as a hallmark of subitizing (e.g., Allik & Tuulmets, 1991; Dehaene & Changeux, 1993; Mandler & Shebo, 1982; van Oeffelen & Vos, 1982; Vos, van Oeffelen, Tibosch, & Allik, 1988).

[2] It should be noted, however, that the models that Balakrishnan and Ashby (1991) rejected presupposed a subitizing range of 1–4 items, whereas visual examination of their data suggested a better fit with a subitizing range of up to 3 items.

A Neuropsychological Approach to Subitizing

Most of the subitizing controversy crystallizes on the issue of whether small displays are quantified by a serial countinglike process or by a spatially parallel process. It is well known, however, that chronometric data are often ambiguous in separating serial and parallel models (Townsend, 1990). Neuropsychological data, on the other hand, are often unique in revealing dissociations of psychological processes that work in complete synergy in the normal adult (e.g., Shallice, 1988). If subitizing and counting were separable psychological processes, then it might be possible to find their occasional dissociation in brain-lesioned patients. Naturally, a selective impairment of subitizing, with preservation of counting, might be difficult to recognize. Such a condition might result only in a slight slowing down of quantification, particularly for sets of 1–3 items. Only fine chronometric techniques might then reveal a deficit, which might not be obvious even to the patients themselves.

In the present work, we looked for the converse deficit, namely impaired counting with preserved subitizing. Counting difficulties are relatively common in brain-lesioned patients (e.g., Holmes, 1918; McFie, Piercy, & Zangwill, 1950; Seron et al., 1991; Warrington & James, 1967). If subitizing was a process radically different from counting, then such patients, despite gross errors of counting, might be expected to quantify small sets of items with both speed and accuracy. There should therefore be a sharp discontinuity in quantification performance between, say, 3 and 4 items.

Counting is a complex process, and there are many ways in which it might be impaired in brain-lesioned patients. For instance, the patients might have difficulty isolating objects from the background, reciting the verbal labels, or memorizing the items already counted. Most relevant to the subitizing–counting debate, however, are patients whose counting errors are due to an impaired serial visual exploration. If subitizing was preserved despite severe difficulties in serial scanning, this would suggest that subitizing is a parallel process that does not require the serial orientation of attention to each item.

Accordingly, all 5 patients included in the present study suffered from complex visual–attentional deficits. Four of them suffered from a parietal lesion or hypometabolism. Parietal lesions, particularly in the right hemisphere, are known to affect attention orienting (Posner, Walker, Frierich, & Rafal, 1984) and counting (McFie et al., 1950; Warrington & James, 1967). Furthermore, all patients showed clinical signs of simultanagnosia. Simultanagnosia is a deficit of the visual perception of complex scenes, with preserved recognition of individual objects (Balint, 1909; Coslett & Saffran, 1991; Hécaen & Ajuriaguerra, 1954; Holmes, 1918; Luria, 1959; Luria, Pravdina-Vinarskaya, & Yarbuss, 1963; Rizzo & Hurtig, 1987; Rizzo & Robin, 1990; see Farah, 1990, for review). Simultanagnosic patients fail to perceive the visual scene as a whole and report only some of its elements. In the more extreme cases, such patients report seeing only one object when presented with two or more of them (e.g., Coslett & Saffran, 1991; Kinsbourne & Warrington, 1962, 1963; Levine & Calvanio, 1978; Luria, 1959). Simultanagnosia is to be distinguished from "shaft vision," however, because the patients are not blind to the rest of the scene. For instance, Luria et al.'s (1963) patient, when "looking out of the window of a car, [. . .] was able to see only one car, then a second, then a third, but always one at a time ('I know there are many but I only see one.')" (p. 222). Coslett and Saffran (1991) found that identification of two briefly presented words (e.g., NEWS PAPER) improved when these could be combined into a single compound word, indicating that "both items in the display must be processed to a fairly high level" (p. 1536).

Not surprisingly, simultanagnosic patients often have difficulties in counting. They have trouble scanning a display and tend to miss some of its elements or to count or report the same elements several times (e.g., Coslett & Saffran, 1991; Hécaen & Ajuriaguerra, 1954; Luria, 1959). Thus one patient, when presented with a picture of two men talking on a verandah, "immediately said 'Here are some people'. When asked to specify the number of people, he pointed first to the head of the men, saying 'one', then to the arm of the other, saying 'two'. He then pointed to the head of the same man, saying 'Here's a third'." (Luria, 1959, p. 446). This particular example, as well as an experiment by Coslett and Saffran (1991), suggests that the counting impairment may extend to the subitizing range. In both cases, however, counting was tested with complex pictures or displays that may have prevented the use of subitizing. We assessed quantification performance with the standard displays used in previous subitizing experiments with normal adults.

Logic of the Present Experiment

In the experiment reported here, the subitizing and counting abilities of 5 brain-lesioned patients were analyzed individually.[3] Quantification was systematically assessed in the range from 1 to 6 items, using both accuracy and response time measures. It was predicted that if subitizing and counting were distinct psychological processes, then some of the patients might show excellent quantification performance up to some limit n, and a sudden onset of counting difficulties for numerosities of $n + 1$ and higher. Because the patient's counting deficits might be manifest only at a short duration of presentation, two different durations of presentation of the visual display were used (200 ms vs. unlimited, response-terminated presentation).

The quantified sets were made of 1 to 6 small rectangles that were either placed randomly in the visual field, or that formed reproducible canonical patterns similar to those found on a dice. Mandler and Shebo (1982) observed that in

[3] The single-patient approach was critical here, because the patients differed considerably in their preserved quantification abilities. Averaging across patients would have smeared the sharpness of the dissociation between subitizing and counting (see McCloskey, 1993, for further discussion of the pitfalls of group studies in neuropsychology).

normal adults, quantification time did not differ for random and canonical patterns until the number of items reached 4 or more. They suggested that subitizing was based on the recognition of the invariant spatial configurations formed by sets of up to 3 items (e.g., 3 = a triangle). Comparison of the patients' quantification of random versus canonical sets should enable us to test this model. Suppose that a patient could still subitize small sets of items (random or canonical) but showed impaired recognition of equally regular but more numerous canonical patterns (e.g., a square made of 4 dots). This would indicate that subitizing and pattern recognition processes are dissociable, and would therefore refute the notion that subitizing is based on the recognition of geometric invariants.

All patients were also tested with visual feature and conjunction search tasks (e.g., Treisman & Gormican, 1988). Visual search for color and for orientation was used to study the extent to which parallel preattentive visual processing—the so-called "pop-out" effect—was intact. Visual search for a conjunction of color and orientation was used to assess the patients' ability to scan a visual display. Serial search and counting are likely to share many cognitive components, including disengaging, moving, and engaging of attention and gaze across the display (Posner et al., 1984); memory for previously scanned locations; and recognition that exhaustive scanning is complete. The patients' pattern of performance in serial visual search might therefore help in understanding the nature of their counting deficit.

General Method

Quantification Tasks

A set comprising between 1 and 6 small rectangles was presented visually on an NEC multisync 3D color screen, controlled by a Toshiba T-5200 portable computer. Subjects were told to say aloud, as fast as they could, the number of items in the set (the instructions avoided the use of the verb "to count"). The microphone was connected to an OROS AU-22 digital board programmed to function as a voice key, which recorded naming time to the nearest millisecond. Responses were digitized at 8 kHz and stored on disk. On subsequent analysis, the experimenter could replay each trial and score each response as correct or incorrect. On occasional trials on which the voice key had been triggered by a spurious noise, response time was either corrected accordingly, or the trial was discarded if the subject's response was not sufficiently distinct (this generally affected less than 5% of responses).

The rectangles to be counted subtended 2.6 mm × 2.3 mm, or an angle of about 18 minutes × 15 minutes at the viewing distance of approximately 50 cm. They were generally white on a dark background, although in two patients (G.O.U. and S.T.E.) we occasionally used a markedly different color for each rectangle in an attempt to make them more discriminable for counting (see below). The stimuli were presented within a 45 mm × 45 mm square area (about 5° of visual angle), centered on the screen, the boundaries of which were not visible and not known to the subject. In the random condition, the stimuli appeared at random locations within the presentation area, with the constraint that the centers of any two rectangles were at least 7.5 mm (51 minutes) apart along either the horizontal or the vertical axis. In the canonical condition, the same "canonical" spatial pattern was used each time a particular numerosity was presented: 2 rectangles were always horizontally aligned, 3 rectangles were organized in an upward equilateral triangle, and the patterns for 4, 5, and 6 rectangles were as they appear on a dice (see Figure 2). The location and size of these canonical patterns were varied from trial to trial, with the constraint that all dots remain within the 45 mm × 45 mm presentation area. Subjects were familiarized with the canonical patterns beforehand. They were told to try and use the additional information provided by the constant shape to improve their quantification of the stimuli.

The random and canonical sets of dots could be presented in two modes. In the unlimited mode, the set remained on until 1,500 ms after the subject had responded. In the 200 ms mode, the set was presented for only 200 ms and then erased. In both cases, stimulus presentation was synchronized with the refresh cycle of the display (60 Hz), and there was a 3-s blank period between trials. Each testing block consisted in 6 presentations of each numerosity from 1 to 6, preceded by 6 warm-up trials and as many untimed training trials as the subject wished.

Visual Search Tasks

A set comprising from 1 to 6 vertical or horizontal colored rectangles (red or green on a black background) was presented using the same apparatus as above. Subjects had to detect the presence or absence of a single target, which was always a red horizontal rectangle (7.5 mm × 1.1 mm or 51 minutes × 7 minutes). Response time were recorded with a 1-ms precision through two large Morse keys connected to the computer. Subjects had to press the right-hand key when the target was present and the left-hand key when it was absent.

Three conditions were compared. In the color feature condition, all distractors were green horizontal rectangles, and the target was therefore singled out by its red color. In the orientation feature condition, all distractors were red vertical rectangles, and the target was therefore singled out by its horizontal orientation. Finally, in the conjunction condition, half of the distractor rectangles were red and vertical, and the other half were green and horizontal. The target was therefore defined only by a conjunction of color and orientation.

In the initial presentation, each trial consisted of a centered yellow fixation cross for 500 ms, followed by a 200 ms blank. Then the rectangles were presented, distributed pseudorandomly in an imaginary rectangle (83 mm × 66 mm, or about 9° × 7°) centered on screen. The set remained on until 900 ms after the subject's response, and was then blanked for 2,000 ms before the

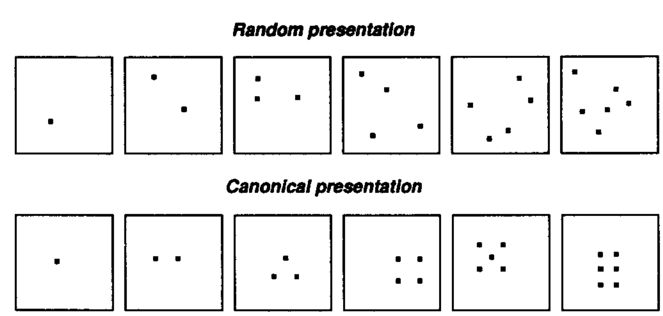

Figure 2. Typical random and canonical sets presented for quantification.

next trial began. Each test comprised an unlimited number of training trials, followed by 24 experimental trials (2 presentations of each Set Size 1–6 with target absent and with target present).

Subject Recruitment and Clinical Testing

All 5 patients were recruited and tested at the Hôpital de la Salpêtrière in Paris, where they were routinely treated. All were chronic cases and were tested at least 1 year postonset. They were selected solely on the basis of the occurrence of clinically identified simultanagnosia, as defined by piecemeal or incomplete description of complex visual scenes together with normal recognition of individual objects. Three patients suffered from unilateral or bilateral lesions of parietal, occipital, and posterior temporal regions. In the other 2 patients, only subcortical lesions were visible, in particular in the caudate nucleus, although in one of them positron emission tomography revealed a bilateral parietal hypometabolism. Further details about each patient are provided below.

A control group of 5 subjects who had no known neurological disorders was also tested for comparison. Each control subject was matched to a patient in sex and age. In subitizing, the controls participated only in the 200-ms presentation mode; because their performance was already close to ceiling, the unlimited mode was not used. In random and canonical subitizing, as well as in the three visual search tasks, the controls took a number of trials equivalent to three of the test sessions used with patients, but grouped within a single experiment.

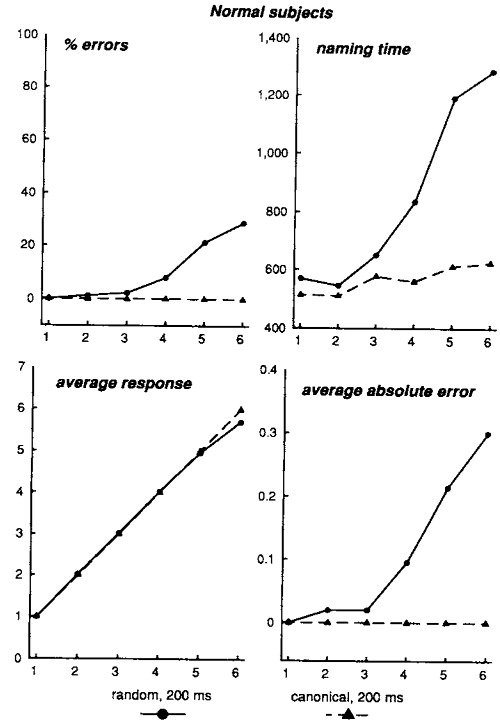

Figure 3. Performance of control subjects in quantifying random and canonical sets of one to six items presented for 200 ms. The percentage of errors started rising for four or more items. Naming time also increased sharply and diverged for the random and canonical presentations between three and four items (the deviation from linearity for six items could be due to knowledge that numerosity never exceeded six). The average response of the subjects did not deviate from the diagonal, indicating an absence of bias. Finally the average absolute error started to be measurable at four and increased linearly above that point.

Results and Discussion for the Control Group

Quantification and visual search paradigms have been extensively studied with normal subjects, and our results basically replicate those of previous experimenters (e.g., Mandler & Shebo, 1982; Treisman & Gelade, 1980). However, we introduce here graphical and statistical methods that were designed specifically to analyze the performance of individual subjects.

Quantification Tasks

Results

Figure 3 displays the four variables that were used to measure quantification performance: percentage of errors, median naming time, average response, and average absolute error. Each variable was computed separately for each subject, each level of numerosity between 1 and 6, and each experimental condition. For simplicity, Figure 3 presents averages over the 5 control subjects; later, the results from each patient are plotted and analyzed individually.

With normal subjects, only two conditions were run: random 200 ms presentation and canonical 200 ms presentation. Normal subjects made no errors whatsoever with canonical patterns. With random patterns, however, errors began to occur relatively often when four or more items were presented. There was considerable variability in normal subjects' error rates in the random condition, especially with large numerosities (e.g., with 6, one subject made no error whereas another made 66.7% errors). In patients, chi-square values were used to compare error rates to consecutive numerosities (e.g., 3 vs. 4), in search of sharp performance discontinuities that might be indicative of a neuropsychological dissociation. In normals, however, errors were generally too infrequent to allow for such an analysis.

Aside from error rate, two variables were used to describe a subject's error pattern. The average response was the average number proposed in response to a particular numerosity. Deviations from the diagonal indicated if the subject tended to systematically underestimate or overestimate numerosity. In the controls, no systematic bias was perceivable, except for numerosity 6 where all erroneous responses fell below the correct value, probably due to the subject's recognition that the presented numerosities never exceeded 6. The average absolute error gave the average absolute deviation of the subject's responses from the true numerosity. A value of 0 indicated perfect quantification. With normal subjects, the average absolute error remained very low from numerosities 1, 2, 3, and started to increase linearly with increasing numerosity from 4 to 6 (Figure 3), perhaps suggesting a discontinuity between 3 and 4.

A final measure of quantification performance was the median naming time. Naming time was analyzed in individual repeated measures analyses of variance (ANOVAs) with numerosity and condition as factors. Both correct and erroneous responses were included in these analyses, because patients sometimes made close to 100% errors in some conditions. In all 5 control subjects, these ANOVAs disclosed massive effects of numerosity, condition, and their interaction (all $ps < 10^{-4}$). Naming time increased significantly with numerosity in both conditions (all $ps <$ .001), but more steeply in the random than in the canonical condition (slopes of 163 ms vs. 24 ms; see Figure 3). More importantly, the curves for these two conditions remained parallel up to a numerosity of 3, and suddenly diverged for numerosities of 4 or more (replicating Mandler & Shebo, 1982). The divergence point was identified statistically as an effect of condition on the reaction time (RT) difference between two consecutive numerosities. There was some interindividual variability in the divergence point, which fell between 2 and 3 for one subject, between 3 and 4 for three subjects, and between 4 and 5 for the remaining subject.

Discussion

Our results with normal subjects replicated earlier findings (e.g., Mandler & Shebo, 1982). In the random condition, numerosity naming time did not increase linearly with numerosity, but showed a marked increase (and a divergence from the canonical condition) only for numerosities of 4 or more. Errors also started to be noticeable at about the same point. These results are suggestive of the existence of two quantification processes, one very accurate for 1, 2, and 3 (subitizing) and the other whose accuracy decreases in proportion to numerosity (counting). However, as noted in the introduction, it is difficult to reject the possibility that subjects were counting throughout the 1–6 range, but that they were faster and more accurate with small numerosities because those placed less demand on their counting process. There was no indisputable discontinuity in the curves of Figure 3 (aside perhaps from the average absolute error). As we shall see below, discontinuities are often much more striking in patients' data.

Visual Search Tasks

Results

The overall error rate did not exceed 4.6% (mean = 2.3%). Individual RTs from each normal subject, whether correct or incorrect, were analyzed in a repeated measures ANOVA with numerosity (1–6), response (absent–present), and condition (color feature, orientation feature, or conjunction) as factors. Four out of 5 subjects showed the classical pattern of linearly increasing RTs with numerosity in the conjunction condition (all $ps < .007$), but not in the two feature conditions (Figure 4). The remaining subject had flat RTs in the color feature condition, but linearly increasing RTs in both the orientation feature and the conjunction condition (both $ps < .008$). As is common in visual search tasks, "present" responses were generally faster than "absent" responses ($p < .001$ in 4 out of 5 subjects). However, this did not interact with a linear contrast on numerosity, and the regression slopes for RT as a function of display size were almost identical for absent and present trials (44 vs. 42 ms/item). This replicated Pashler's (1987) observations that a 2:1 ratio of absent-to-present slopes does not necessarily obtain for small display sizes.

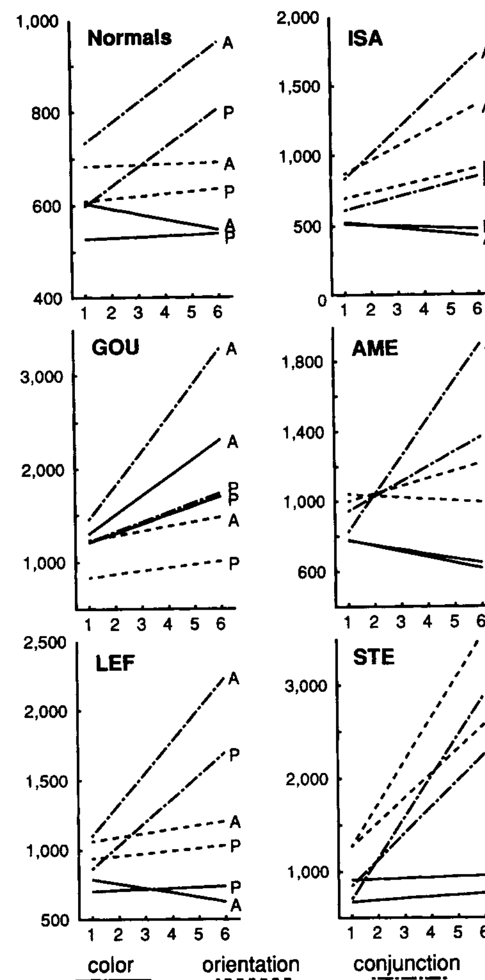

Figure 4. Reaction times (in milliseconds) of control subjects (normals) and of the five patients (I.S.A., G.O.U., A.M.E., L.E.F., and S.T.E.) in visual search for color, orientation, or a conjunction of both, as a function of display size (one–six items). A = absent trials, P = present trials. Different scales were used for each graph.

Discussion

As expected, a pop-out effect was observed in color search. Orientation search was more ambiguous, with at least one subject exhibiting a clear increase of RTs as a function of set size in this condition. In 4 out of 5 subjects, orientation search appeared slower than color search. The apparently higher difficulty of the orientation task should be kept in mind when analyzing the performance of brain-lesioned patients. Finally, a pattern of RTs compatible with serial search was observed in the conjunction condition. RT increased linearly with display size. The slope ratio of absent to present trials was closer to 1:1 than to 2:1, which is the value predicted by a simple self-terminating search process (cf. Pashler, 1987). However, the same stimuli were used in a previous study of visual search in normal subjects (Dehaene, 1989), in which a 2:1 ratio was obtained for larger display sizes. This study concluded that the 1:1 effect could be due to a variety of RT deformations for small display sizes (for instance, a strategy of double-checking in "absent" trials) and therefore was not incompatible with a serial search model. Most models of visual search, even if they depart from the specific assumptions of the Treisman and Gelade (1980) model, still incorporate a process of serial exploration and tagging of the visual scene.

The search rate averaged across the 5 subjects was 44 ms/items in conjunction search, a value much smaller than the mean increase of 271 ms/item observed in the quantification task over the range 3–5. Assuming that the subjects were counting in the latter condition, this suggests that counting time in normal subjects is not primarily determined by the time taken to move attention across the display, or to keep track of previously scanned items, because these processes are supposedly common to counting and to visual search. The counting rate rather seems related to the mental recitation of the series of numbers. As we shall see below, the converse seems to hold for simultanagnosic patients.

Patient I.S.A.

Case Report

I.S.A. was a 34-year-old right-handed woman. While she was pregnant, she was admitted to the hospital with severe headache and hypertension. She presented left homonymous hemianopia (blindness in the left visual field), left visual and motor neglect, and constructional apraxia. Computed tomography (CT)-scan revealed a right parieto-occipital hemorrhage. Two months later magnetic resonance imaging (MRI) showed, in addition to the sequelae of the hemorrhage, several additional bilateral ischemic lesions in the subcortical white matter and corpus callosum. This aspect was identical 1 yr after the initial episode, when testing was carried out. At that time, the patient had prosopagnosia and complained that she perceived the shape of visual objects with some distortion. She easily identified normal and unusual views of single objects. Color naming, writing, and drawing were satisfactory. There was left-ear extinction on dichotic listening. Goldmann perimetry showed partial left homonymous hemianopia affecting essentially the inferior quadrant. The patient had difficulties apprehending a scene at a glance and gave piecemeal descriptions of complex pictures. As described above, MRI showed a cortical lesion affecting Areas 19, 37, and 39 on the right, in addition to bilateral subcortical lesions.

Experimental Testing

In quantification, patient I.S.A. participated in two tests in the random 200 ms condition, one test in the random unlimited condition, and one test in the canonical 200 ms condition. In visual search, she participated in two tests in each of the three conditions.

Quantification Tasks

Results. The patient made 43.1% errors (31/72) in the random 200 ms condition, but made no error whatsoever in the random unlimited and canonical 200 ms conditions (χ^2 [2] = 39.0, $p < .0001$; see Table 1). Figure 5 shows that the errors were not randomly distributed across the six numerosities, χ^2 [5] = 48.5, $p < .0001$. I.S.A. made very few errors in quantifying random displays of 1, 2, or 3 items, but she erred systematically with random displays of 4, 5, or 6 items. The rate of errors increased suddenly; she made only 8.3% errors (1/12) with 3, but 75% errors (9/12) with 4, χ^2 (1) = 11.0, $p < .001$. In 96.8% (30/31) of her errors, she underestimated the correct numerosity. The larger the presented numerosity, the more her responses deviated from correctness, as demonstrated by a significant increase of the average absolute error with numerosity, r^2 (29) = 17.4%, $p < .02$ (see Figure 5, bottom).

The analysis of numerosity naming times also suggested that I.S.A. applied separate strategies to numerosities below 3 as opposed to these above 4. In all three conditions of stimulus presentation, response time increased with numerosity ($p < .0001$). However, the rate of increase differed widely across conditions, as shown by a significant interaction of condition and a linear contrast for numerosity, $F(2, 125) = 92.5$, $p < .0001$. RT increased by only 38 ms/item in the canonical 200 ms condition, by 110 ms/item in the random 200 ms condition, and by as much as 394 ms/item in the random unlimited condition. The latter value strongly suggests that I.S.A. processed the items serially and attempted to count them. However, this linear increase of RTs was not manifest over the entire range of numerosities. There was no increase in RT when going from 1 to 2, and only a small increase from 2 to 3. The curves for the three conditions were parallel in the 1–2 range, $F(2, 42) = 0.98$, *ns*, and in the 2–3 range, $F(2, 42) = 0.93$, *ns*, but diverged

Table 1
Performance of the 5 Patients in Quantification Tasks (Percentage of Errors)

Condition	I.S.A.	A.M.E.	G.O.U.	L.E.F.	S.T.E.
Random					
200 ms	43.1	52.9	47.6	52.2[a] 38.7[b]	44.0
Unlimited	0.0	30.6	40.3	64.8[a] 24.3[b]	34.1
Canonical					
200 ms	0.0	12.9	28.1	20.3	43.9
Unlimited	—	—	4.3	—	9.1
6 colors					
200 ms	—	—	45.1	—	—
Unlimited	—	—	—	—	15.0

Note. Dashes indicate conditions that were not run with a given patient.
[a] Unknown range (before the patient realized that 6 was the highest numerosity ever presented in the experiment). [b] Known range.

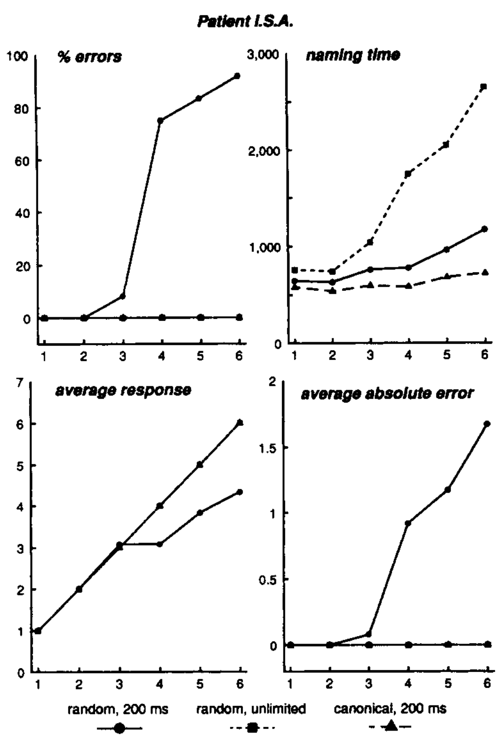

Figure 5. Quantification performance of patient I.S.A. In the random 200-ms condition, a discontinuity appeared between three and four items.

when going from 3 items to 4 items, $F(2, 42) = 13.68, p < .0001$. Thus, both RTs and errors gave evidence of a processing discontinuity between 3 and 4.

Discussion. I.S.A. was fast and accurate at quantifying sets of up to 3 items. She became very slow with sets of 4 or more randomly organized items, and her response times were then consistent with a counting process. If display presentation was unlimited, she managed to count flawlessly, albeit very slowly, up to 6 items. However, if the stimuli were presented for only 200 ms, she made a large number of errors of underestimation, perhaps missing some of the targets. Finally, it should be noted that I.S.A. was extremely fast and accurate when the set was presented in canonical form. Examination of her RTs suggested that, just like a normal subject, she did not have to count to recognize a canonical pattern of 4, 5, or 6 items. In a nutshell, a clear-cut dissociation was found between preserved subitizing for numerosities up to 3, preserved recognition of canonical forms, and impaired serial counting with systematic underestimation.

Visual Search Tasks

Results. I.S.A.'s visual search performance was virtually flawless. She made 4.2% errors (3 misses, 1 false alarm) in the two feature search tasks and no errors in conjunction search. In the color feature search, her response times did not vary with display size, $F(1, 36) = 2.94, .05 < p < .10$), indicating a normal pop-out (see Figure 4). However, such was not the case for the orientation feature search, in which RT increased significantly with display size, $F(1, 36) = 13.0, p < .001$. The observed slopes of 42 ms/item and 100 ms/item respectively, for present and absent responses were almost in a 2:1 ratio, suggesting the use of a serial self-terminating strategy. There was in fact no significant difference between orientation search and conjunction search, in which RT also increased with display size, $F(1, 36) = 5.42, p = .026$, and in which the slopes for present and absent responses were respectively 47 ms/item and 180 ms/item. I.S.A.'s mean RTs and slopes were significant slower than the controls' in both orientation search and conjunction search (all $ps < .005$), but not in color search.

Discussion. Comparison with the control subjects suggests that parallel processing of color was normal in patient I.S.A. Her lack of a pop-out effect for orientation was compatible with her complaints of distortions of the visual field, but it could also be due to the higher difficulty of the orientation feature task. More importantly, I.S.A. was impaired in the conjunction task. Her search rate was approximately three or four times slower than that of a normal subject. This could not be attributed to a global cognitive slowing down, because her RTs in the color feature condition were almost identical to the controls'. I.S.A.'s difficulties in serial exhaustive scanning of a visual display may explain her counting impairment. With short stimulus presentation, she did not have enough time to complete the exploration of the display, and therefore reported a too small number. Given more time, however, she was able to quantify accurately all the sets. Conversely, her perfect performance in the subitizing range of 1–3 items despite her serial search difficulties suggests that subitizing does not require serial scanning and therefore is a parallel process.

Patient A.M.E.

Case Report

A.M.E. was a 51-year-old right-handed woman. At the age of 46, she suffered a severe transient hypotension, possibly following inadequate treatment for hypertension. As a consequence of this episode, she became apathetic and indifferent. In addition, she had severe dysgraphia, constructive apraxia, and some reading and calculation difficulties. These deficits remained essentially unchanged until testing was carried out, 5 yr later. At this time, she produced piecemeal descriptions of complex pictures and had an abnormal performance on letter or shape cancellation tests (14/60 and 19/22 missed targets, respectively). Her visual field was normal on Goldmann perimetry, as was her visual acuity. MRI revealed a bilateral lesion affecting the occipital and posterior parieto-temporal region, corresponding to part of Brodmann's Areas 19, 37, 39, and 7 of both hemispheres. Single photon emission computed tomography (SPECT) showed bilateral hypometabolism in the posterior temporal and parietal regions.

Experimental Testing

In quantification, A.M.E. participated in two tests in each of the random 200 ms, random unlimited, and canonical 200 ms conditions. In visual search, A.M.E. participated in one test in each condition.

Quantification Tasks

Results. A.M.E. made significantly less errors in the canonical condition (12.9%) than in either the unlimited or 200 ms random conditions (30.6% and 52.9% errors, respectively; both *p*s $< .013$ on χ^2 tests; see Table 1). The pattern of errors was particularly clear in the random 200 ms condition (see Figure 6): Error rates differed significantly across the six numerosities (χ^2 [5] $= 16.8$, $p < .005$), and there was a sudden increase in errors between 2 and 3 items (8.3% vs. 75.0% errors; χ^2 [1] $= 11.0$, $p < .001$). Most errors (32/36 = 88.9%) were underestimations whose amount did not appear to vary consistently with numerosity. A similar pattern was observed in the random unlimited condition, although it did not reach significance owing to the small number of observations.

The RT analysis confirmed the presence of a processing discontinuity between 2 and 3 items. In all three conditions, RTs increased with numerosity ($p < .0001$), but at significantly different rates ($p < .0001$): 83 ms/item in the canonical 200 ms condition, 201 ms/item in the random 200 ms condition, and 318 ms/item in the random unlimited condition. In the latter case, however, RTs did not vary linearly with numerosity, but jumped from about 900 ms for displays of 1 or 2 items, to more than 2,000 ms for displays of 3 or more items (Figure 6). The curves for the three conditions were parallel in the 1–2 range, $F(2, 63) = 1.55$, *ns*, but diverged sharply when going from 2 to 3 items, $F(2, 64) = 10.1$, $p = .0002$.

Discussion. Patient A.M.E. showed a major processing discontinuity similar to I.S.A.'s, with preserved quantification of small displays as compared to larger ones. In A.M.E.'s case, the processing discontinuity was found between 2 and 3 items. A.M.E.'s counting deficit was apparently more severe than I.S.A.'s; it could not be completely compensated even when the displays were presented for an unlimited amount of time. Even the processing of canonical displays, although significantly better, was slightly slow and error prone as compared to normal subjects.

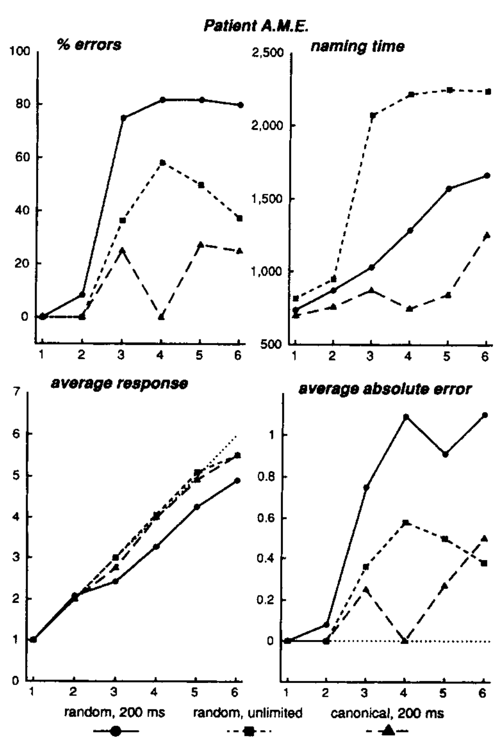

Figure 6. Quantification performance of patient A.M.E., showing a discontinuity between two and three items.

Visual Search Tasks

Results. A.M.E.'s performance was quite good: 0% errors in color search, 16.7% errors in orientation search, and 4.2% errors in conjunction search (all were target misses). Her response times also presented a normal pattern. In the two feature search conditions, RT did not increase with display size (both *p*s $> .05$). In conjunction search, however, the increase was significant, $F(1, 12) = 10.5$, $p = .007$, and the slope ratio of the slopes for absent versus present trials was close to 2:1 (216 ms/item vs. 86 ms/item), suggestive of serial self-terminating search. A.M.E. was significantly slower than the controls in both orientation search and conjunction search ($p < .001$) but not in color search ($p > .05$).

Discussion. A.M.E. was only mildly impaired in visual search: she made few errors but was significantly slower than normal subjects. Because she was slow in both feature and conjunction search, this may have reflected the negative effects of any brain lesion on response time, rather than a specific attentional deficit. Nevertheless, it is tempting to view her difficulties with counting and with visual search as manifestations of a single underlying deficit of serial exploration. We shall suggest below that she suffered from a deficit in using spatial cues to systematically and exhaustively explore a visual display. Contrary to normal subjects, she could not use spatial location to tell whether a given item had been previously explored. Given that all 5 patients in the present study exhibited a similar pattern of impairment, however, this issue is best delayed until the General Discussion section.

Patient G.O.U.

Case Report

G.O.U. was a 47-year-old right-handed man who suffered an occipital bullet injury at the age of 18. The bullet was removed surgically. In addition to occasional seizures, the patient experienced stable visual impairment consisting essentially in a left macular homonymous scotoma associated with some degree of spatial disorientation and simultanagnosia. He mentioned that when exploring large and complex displays, such as a cinema screen or a crowded room, he tended to remain stuck to fragments or details of the scene, with ensuing difficulties in grasping the meaning of the whole. Once his visual fixation was detached from a given object, he had difficulties finding this object again in space. MRI showed a wide enlargement of the posterior part of the lateral ventricles, associated with abnormal cortical signal intensity affecting Areas 17, 19, and 37 bilaterally, and Areas 39 and 40 on the left.

Experimental Testing

In quantification, G.O.U. participated in four tests in the random 200 ms condition, and in two tests in each of the random 200 ms, canonical 200 ms, 6 colors 200 ms, and random unlimited conditions. In visual search, G.O.U. participated in two tests in each of the three conditions.

Quantification Tasks

Results. G.O.U.'s percentage of errors in each of the five quantification conditions appear in Table 1. He was fairly accurate only in the canonical unlimited condition (4.3% errors). In all other conditions, he made many errors that were not randomly distributed across numerosities (all $ps < .006$ on χ^2 tests). Performance was always extremely accurate up to 2 items, at which point he suddenly made a large number of errors (Figure 7). For instance, in the random 200 ms condition, he made no errors with sets of 1 or 2 elements (0/23 and 0/24 errors, respectively), but 62.5% errors (15/24) with sets of 3 elements (χ^2 [1] = 21.8, $p < .0001$). Most errors (56/68 = 82.4%) were underestimations of the true numerosity, and the average absolute error increased with numerosity.

The sole exception to this discontinuity between 2 and 3 in errors rates was in the random unlimited condition. Here his performance was virtually flawless up to three elements (1/32 = 3.1% errors), but suddenly degraded with four elements (7/10 = 70.0% errors). His surprisingly good performance with three items was most likely due to serial counting, however; his median response time to three items was 1,840 ms and well into the linearly increasing part of the RT curve (see Figure 7).

In all five conditions, RTs increased with numerosity ($p < .0001$), with rates ranging from 66 ms in the 200-ms canonical condition to 849 ms in the random unlimited condition. The rate of 179 ms/item in the canonical unlimited condition suggested that G.O.U. attempted to count even with canonical patterns. In the random unlimited condition, there was a significant 358 ms increase in RT even between 1 and 2 items, $F(1, 21) = 6.82$, $p = .016$. In all other conditions, however, RTs did not increase significantly between one and two items and diverged only between two and three items.

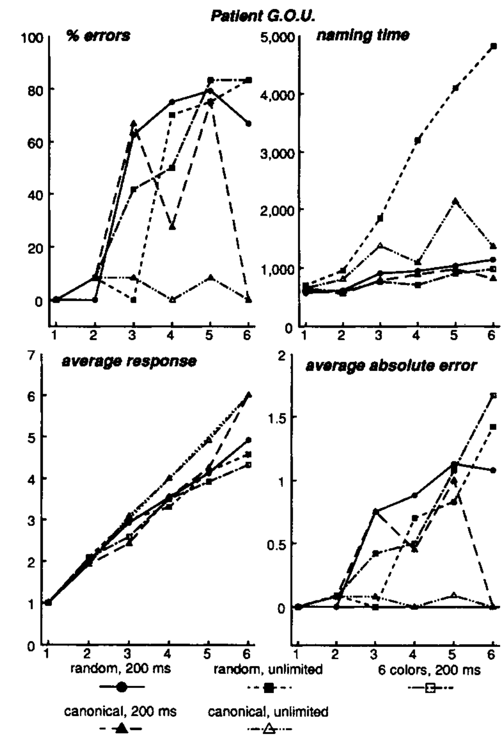

Figure 7. Quantification performance of patient G.O.U. With 200-ms presentation, his performance deteriorated sharply, for both random and canonical display, when three or more items were presented. Given more time, however, he was able to recognize the canonical patterns and to count accurately up to three.

Because G.O.U.'s quantification difficulties could be due to an inability to distinguish the already counted items from the remaining ones, we presented him both with the regular random patterns made of white rectangles (random 200-ms condition) and with random patterns made of rectangles each of a vividly different color (6 colors, 200-ms condition). However, increasing the discriminability of the items by color did not appear to help him much (47.6% vs. 45.1% errors), even though there was some improvement with sets of three or four items (68.8% vs. 45.8%); χ^2 (1) = 3.54, p = .06. G.O.U. might have suffered from an additional deficit of color processing (see the color feature search below).

Discussion. G.O.U. showed a sharp dissociation between preserved quantification of displays of one or two items, and impaired counting with displays of three items or more. His performance was quite similar to A.M.E.'s, showing a counting impairment even in conditions of unlimited display duration. His counting was also extremely slow, up to almost 1 s per item in the random unlimited condition.

Contrary to the previous two patients, G.O.U. showed a highly deteriorated performance with canonical patterns, and his response times suggested that he had to quantify them by counting rather than by some form of rapid pattern recognition. His impaired recognition of canonical patterns suggests that pattern recognition routines were probably not responsible for his preserved subitizing of random sets of 1 and 2 items. The dissociation between impaired pattern recognition and preserved subitizing goes against the notion that subitizing consists in recognizing the regular geometric patterns formed by most sets of 1, 2, or 3 objects (e.g., Mandler & Shebo, 1982). Because other cases are also relevant to this point, however, we shall address it more fully in the General Discussion section.

Visual Search Tasks

Results. G.O.U.'s performance was adequate in visual search. He made 10.4% errors (4 misses, 1 false alarm) in the color feature condition, no errors in the orientation feature condition, and 6.3% errors (3 misses) in conjunction search. In the orientation feature search, RT did not increase significantly with display size, $F(1, 36) = 2.72$, *ns*), suggesting a preserved pop-out for orientation. Color did not pop-out; RTs increased with display size, $F(1, 36) = 10.1$, $p < .005$, at a rate of 99 ms/item for present responses and 205 ms/item for absent responses.[4] Search was also slow and serial in conjunction search, $F(1, 36) = 24.1$, $p < .0001$, where the rates were 105 ms/item for present responses and as much as 369 ms/item for absent responses. G.O.U. was significantly slower than controls in all three conditions of visual search, all $ps < .001$.

Discussion. Like Patients I.S.A. and A.M.E., G.O.U. showed a drastic reduction of the speed of visual search. Some degree of parallel processing remained present, at least in the orientation pop-out condition, but serial search was extremely slow. It seems likely that this visual exploration deficit was responsible for his counting difficulties. Again, however, quantification of small sets of up to 2 items appeared unaffected, suggesting the use of a preattentive subitizing procedure.

Patient L.E.F.

Case Report

L.E.F. was a 51-year-old right-handed woman, who suffered a severe hypotensive episode during routine surgery, at the age of 47. She presented initially with aphasia (which disappeared rapidly), apraxia, and complete Balint's syndrome and Gerstmann's syndrome. Testing was carried out 4 yr after onset. Clinical examination showed no sensory, motor, or linguistic deficit. The patient still had severe apraxia, as well as agraphia and acalculia. Her visual field was normal on Goldmann perimetry. She could name objects, famous faces, and colors. However, she still had some difficulties analyzing complex pictures, which she described in a piecemeal and poorly integrated fashion. MRI showed bilateral abnormal signal intensity in the caudate and lenticular nuclei and a few quite small anomalies in the hemispheric white matter. Positron emission tomography using ^{18}Fluorodeoxyglucose showed bilateral hypometabolism in the posterior parietal regions.

Experimental Testing

In quantification, L.E.F. participated in 4 tests in both the random 200 ms condition and the random unlimited condition, and in 2 tests in the canonical 200 ms condition. In visual search, she participated in two tests in each of the three conditions.

Quantification Tasks

Results. L.E.F.'s percentage of errors in each condition appear in Table 1. Her behavior changed radically in the course of testing. Her results were therefore split into two blocks. She initially did not recognize that the maximum number of items that would be presented was 6. Other subjects discovered this in the first few training trials, even though it was not mentioned in the instructions. However, L.E.F.'s counting was so badly impaired that she appeared to believe that up to 13 items were presented. When asked to point toward the items while counting them, she readily pointed two or three times toward each item, not noticing that she had already counted them. Accordingly, she made many errors in both the random 200-ms and random unlimited 200-ms conditions (52.2% and 64.8%). Most of her errors were extreme overestimations of numerosity (e.g., 12 instead of 6).

Eventually, the 6 canonical patterns were introduced to the patient and she inferred that 6 was probably the maximum in the other conditions, too. After that point, she never gave any responses higher than 6, and her error rate for 6 dropped considerably (see Figure 8), although she still made overestimation errors with numerosities 3, 4, and 5. Like patients A.M.E. and G.O.U., she made many fewer errors with 1 and 2 than with higher numerosities. In the unlimited and canonical conditions, there were sharp discontinuities in error rates for 2 versus 3 (e.g., 0% vs. 91.7% errors, $p < .0001$, in the random 200 ms, unknown range condition). With 200 ms presentation, however, L.E.F. made some errors even when 2 items were presented, although she still performed perfectly with sets of one item.

In all conditions, RTs increased with numerosity (all $ps < .042$), with rates ranging from 91 ms/item in the canonical 200 ms condition to 728 ms/item in the random 200 ms, unknown range condition. Except in the canonical condition, there was always a significant increase in RTs when going from 1 to 2 items. However, from 1 to 2, RTs increased in parallel across conditions, $F(4, 121) = 0.55$, *ns*, and they diverged significantly only when going from 2 to 3 items, $F(4, 119) = 8.19$, $p < .0001$.

Discussion. Like Patients A.M.E. and G.O.U., L.E.F. showed a drastic deficit in serial counting, together with relatively preserved quantification of sets comprising 1 or 2 items. On the surface, L.E.F.'s counting deficit appears quite different from the others': she consistently overcounted the presented sets, whereas all other patients underestimated them. These two kinds of difficulties may,

[4] We note in passing that the results of Patients I.S.A. and G.O.U. indicate a double dissociation of pop-out from color and pop-out from orientation.

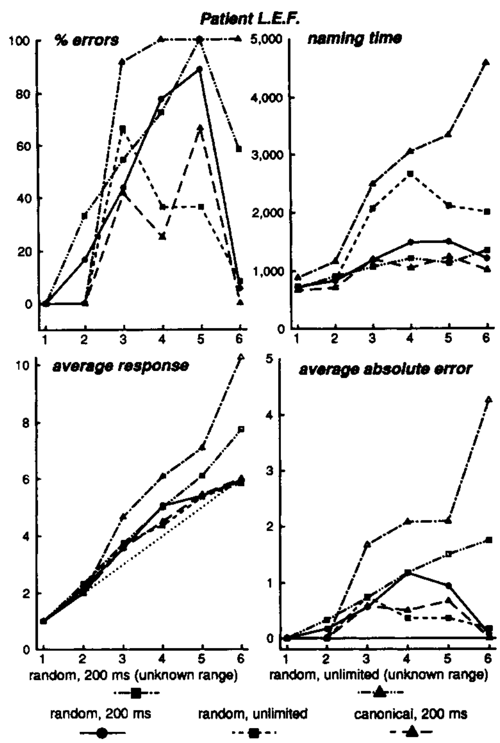

Figure 8. Quantification performance of patient L.E.F. Contrary to other patients, L.E.F. did not immediately realize that the range of numerosities was one–six, and on the first few tests (marked *unknown range),* she widely overestimated numerosity by overcounting the same items. However, numerosities of one and two, especially with unlimited presentations, escaped this systematic pattern of overestimation.

however, originate from the same core failure of keeping track of previously counted items. In fact, L.E.F.'s overt finger counting gave direct evidence for a confusion between counted and yet-to-be-counted items; she overtly counted the same rectangles over and over again without recognizing her errors. This also predicts that she might be abnormally slow in serial visual search, because she might lose time exploring over and over again the same distractors. This prediction was confirmed in the visual search task.

Visual Search Tasks

Results. The patient's performance was excellent: She made no errors in color and conjunction search, and 2.1% errors (1 miss) in orientation search. Pop-out was observed in both color and orientation search, in which RTs did not vary significantly with set size ($p > .33$). Color search times were on the border of the normal range ($p > .05$), whereas orientation search times were significantly slower than in normals ($p < .001$). RTs compatible with serial search were observed in the conjunction condition, with RTs increasing significantly with set size at a rate of 167 ms/item for present trials, and 227 ms/item for absent trials (Figure 4). This rate of conjunction search was markedly slower than in normals (z score = 34.5, $p < .0001$).

Discussion. As expected, L.E.F.'s most salient deficit in visual search, as in all the patients reported here, was a pronounced reduction of scanning speed. Parallel processing of the "pop-out" type seemed preserved, even if slightly slowed. We attribute her slow speed of serial search not to a difficulty in attention orienting, but to a failure in implementing an exhaustive and efficient search path throughout the set of items. She did not seem to discriminate items that she had already explored from items that remained to be explored.

As we discuss more fully in the General Discussion section, the core deficit of simultanagnosia is a failure in using spatial tags to refer to individual objects (e.g., Coslett & Saffran, 1991). Because all the rectangles in our quantification task were physically identical and were distinguished only by their spatial location, it is perhaps not surprising that simultanagnosic patients had difficulties knowing which of them remained to be counted. However, this also suggests that counting performance should improve considerably if we were to differentiate the counted items using features other than spatial location. Our final patient, S.T.E., showed precisely such an improvement when each of the items was presented in a different color.

Patient S.T.E.

Case Report

S.T.E. was a 34-year-old right-handed woman who underwent emergency surgery for a ruptured ectopic pregnancy. On awakening she presented with a severe visual deficit corresponding to an inferior altitudinal amputation of the visual field on Goldmann perimetry (blindness in the lower part of the visual field), associated with Balint's syndrome. Testing was carried out 2 years later. At that time, visual field was normal on Goldmann perimetry. The patient could correctly identify object pictures visually, except for a few perceptual errors. She still showed a visuospatial behavior suggestive of simultanagnosia. She had difficulties grasping the relations between parts of complex displays. Similarly, she copied the Rey-Osterrieth figure by juxtaposing details without perceiving adequately the overall structure. MRI showed abnormal signal intensity in the head of the right caudate nucleus only.

Experimental Testing

In quantification, S.T.E. participated in 4 tests in each of the random 200 ms, random unlimited, and 6 colors unlimited conditions, and in 2 tests in each of the canonical 200 ms and canonical unlimited conditions. In visual search, she participated in 2 tests in each of the three conditions.

Quantification Tasks

Results. S.T.E.'s percentage of errors in each of the five quantification conditions appear in Table 1. She had considerable difficulties with 200 ms presentations, whether the counted set was presented in canonical or in random format (44% errors in both cases). With unlimited presentations, she still made a large number of errors with random patterns but showed a considerable improvement both with canonical patterns and with random patterns composed of rectangles of different colors (both $\chi^2 > 13.5$, $p < .0003$).

With 200 ms presentations, unlike any of the previous patients, S.T.E. made errors even when only 1 or 2 items were presented (Figure 9). There was only weak evidence for a discontinuity between 2 and 3 in error rates to random patterns, χ^2 (1) = 4.81, p = .028. In the random unlimited condition however, S.T.E. was near perfect with 1 and 2 items, and there was a sudden onset of errors between 2 and 3, χ^2 (1) = 8.28, p = .004. Finally, in both the canonical unlimited condition and the six colors condition, the error rate remained very low for sets of up to 4 items. There was a systematic pattern of underestimation in all conditions of unlimited presentation, but not with 200 ms presentation duration.

Examination of the patient's response times helped sorting out the conditions in which she was probably counting (Figure 9). In both the random unlimited and the six colors conditions, RT increased sharply with set size at rates of 1,424 and 1,189 ms/item respectively (both $ps < .0001$).

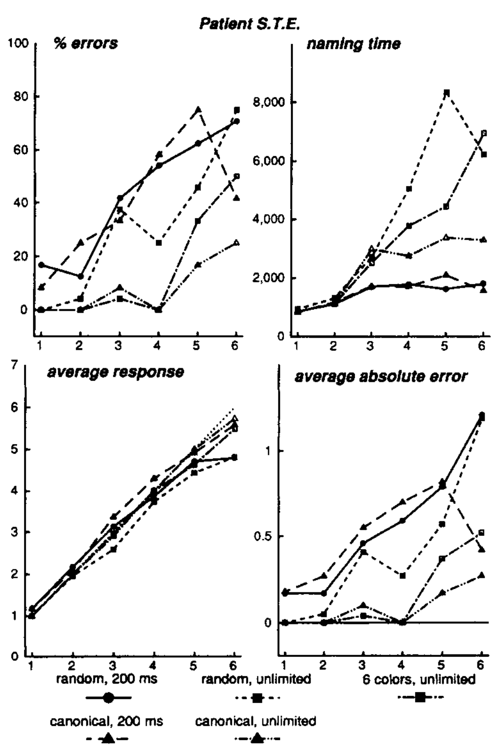

Figure 9. Quantification performance of patient S.T.E. Quantification was faster and more accurate with one and two items than with larger numerosities. However, with 200-ms presentation the patient even made errors on one and two. Performance improved considerably with canonical patterns or when the counted items were each of a different color.

For instance it took S.T.E. between 3.5 s and 5 s to respond to a 4-item display. It therefore seems likely that her good performance with colored displays of 3 and 4 items was due to slow counting. By contrast in all other conditions, RTs showed a similar pattern with (a) relatively little or no increase from 1 to 2 items, (b) a sharp increase from 2 to 3 items (all $ps < .035$), and (c) little or no increase above 3 items. The flatness of RTs for sets or 3 or more items suggests either that S.T.E. did not attempt to count, or that if she did she stopped counting after an approximately constant amount of time regardless of the input numerosity. The fact that sets of 1 or 2 items took significantly less time and were processed significantly better than sets of 3 or more items suggests that quantification of 1 or 2 items proceeded using a different strategy.

Discussion. S.T.E. showed less consistent evidence than previous patients for a processing discontinuity between 2 and 3 items. With short presentations, she occasionally made errors even with sets of 1 or 2 items. She was also slow (almost 1 s for a set of 1 item). Nevertheless, her RTs suggested the use of a fixed and relatively accurate strategy for sets of 1 or 2 items, whereas sets of 3 or more items were often counted painfully slowly and inaccurately.

The case of S.T.E. is more interesting for its indications about the locus of the counting impairment in simultanagnosia. S.T.E. was quite impaired in counting 3 or 4 rectangles of similar size, shape, and color, but her counting accuracy improved dramatically when the rectangles were all of a different color.[5] In this condition she spontaneously reported: "I count the colors, not the dots. There is some green, some red, some white, etc.". This suggests that the counting procedure itself was intact, but that she had difficulties in applying it to sets of items that were distinguished only by their spatial location. We assume that she failed to use spatial tags to keep track of the items that she had already counted. As soon as she could use a nonspatial dimension for that purpose (e.g., color), her counting improved.

S.T.E.'s spatial deficit might be expected to extend to serial visual search tasks, just like other simultanagnosic patients. Lacking spatial cues, she might have difficulties in systematically exploring a visual display. This was verified below.

Visual Search Tasks

Results. S.T.E. made no errors in color search, 10.4% errors in orientation search (3 misses, 2 false alarms), and 10.4% errors in conjunction search (5 misses). Her RTs (see Figure 4) did not increase with set size in color search, $F(1, 36) = .10$, *NS*, suggesting normal pop-out even though she was significantly slower than controls ($z = 3.84$, $p < .001$). By contrast, RTs increased significantly with set size in both orientation search and conjunction search, with

[5] In the six colors condition, S.T.E. started to make errors again with sets of 5 or 6 items. This may be explained by the increasing similarity of the colors as numerosity increased.

average rates of 389 ms/item and 370 ms/item respectively. These rates were abnormally slow compared to those of the controls (both $ps < .0001$).

Discussion. In feature search, S.T.E. showed a pattern similar to I.S.A., with close to normal pop-out in the color condition but RTs consistent with serial search in the orientation condition. In conjunction search, she also appeared to use a very slow and serial search strategy. Once again, we suggest that her slowness was mostly due to the lack of a systematic and exhaustive exploration of space, caused by her inability to keep track of previously explored spatial locations. The fact that she occasionally missed the target seems consistent with this interpretation.

General Discussion

All 5 patients reported here showed relatively preserved quantification of sets of 1, 2, or sometimes 3 items, together with impaired counting of larger sets. In some cases, this dissociation was very sharp, with close to 0% errors on, say, sets of 2 items, and close to 100% errors on sets of 3 items. This shows that subitizing can be preserved when counting is impaired.

Because we did not yet find a patient with the converse dissociation (impaired subitizing with preserved counting), the interpretation of this result is not completely straightforward. Contrary to a double dissociation, a simple neuropsychological dissociation may simply mean that the impaired task is more difficult than the preserved one (see Shallice, 1988)—and in some sense quantification certainly becomes more difficult with increasing numerosity. Before concluding that subitizing and counting are radically different processes, it is therefore necessary to understand better the nature of the patients' counting impairment. In this discussion, we present arguments suggesting that simultanagnosic patients suffer from a general deficit of serial visual exploration due to an inability to use spatial tags to refer to object locations. This implies that their preserved quantification of small displays does not rely on serial visual processes but is based on a qualitatively different parallel process of subitizing.

Deficits of Visual Exploration in Simultanagnosia

All patients reported here suffered from somewhat similar deficits in quantification and visual search tasks. First, serial counting was very slow and error prone. Most patients missed some of the items, whereas one (L.E.F.) systematically counted the items several times. Second, serial visual search was pathologically slow but with few errors, most of which were target misses. In feature search, parallel pop-out was often preserved.

We may quickly dismiss low-level visual deficits as responsible for these difficulties. First, even though two patients (I.S.A. and G.O.U.) suffered from visual field deficits, their fast and accurate performance in visual search, particularly in at least some conditions of feature search, indicated good vision of the stimuli used in our study. Indeed, most patients without neglect can compensate for visual field cuts by reorienting their gaze so that the relevant stimuli fall within intact portions of their visual field. Globally, the patients' performance appeared uncorrelated with the presence of visual field deficits. Second, at least one patient (L.E.F.), far from missing any visual targets, counted them more than once. Third, visual field deficits cannot explain the preservation of the quantification of small sets. If there was a finite probability of missing each of the items, then the patients should occasionally fail to quantify sets of 1 or 2 items. Their performance should decrease continuously as numerosity increases (this is demonstrated formally in the Appendix). No low-level deficit seems capable of explaining the perfect performance of some patients with 2 or 3 items.

Another interpretation of our findings is that the patients' attentional movements were slowed. Posner et al. (1984) have described deficits of the disengagement of attention in neglect patients with unilateral parietal lesions. Indeed, simultanagnosia is sometimes described as a bilateral form of neglect (e.g., Newcombe & Ratcliff, 1989). However, several problems confront this interpretation. First, in those cases of selective simultanagnosia in which Posner et al.'s cuing paradigm was used, attention disengagement, shifting, and engagement were found to be normal (Coslett & Saffran, 1991; Rizzo & Robin, 1990; similar results were obtained with our patient S.T.E.). Second, it is not clear how such a deficit would explain the pronounced difference between counting and visual search performance. The good accuracy of the patients in serial visual search suggests that they were able to explore a visual display (even if slowly) without missing any item. Why then did they obtain such a poor score in counting, when the display was presented for an unlimited amount of time? If attentional movements were simply slowed, counting should have been slow but accurate in the unlimited condition. Yet, 4 out of 5 patients contradicted this prediction.

We therefore believe that the deficit in our subjects lies not in attentional movements per se, but in keeping track of the spatial locations that they have previously explored. When all visual objects are identical and are distinguished only by their locations, counting becomes virtually impossible because the subjects cannot figure out, when they encounter an object, if they have already counted it. Most subjects in our study failed to count some of the items, presumably because they thought that they had already counted them. However, one subject (L.E.F.) appeared to count the same items over and over again, winding up with a large overestimation of numerosity. In this subject, explicit pointing during counting gave direct evidence for a deficit in keeping track of previously counted items.

In visual search, a deficit of object individuation by spatial location should not have consequences as drastic as in counting. If the subject explores the same items more than once, search will be slower, but not necessarily less accurate. The only requirement is that the search be exhaustive, which can be approximately achieved by scanning the display for a sufficiently long time. We note that most of the subject's errors were target misses, which is consistent with the hypothesis that they did not always complete an exhaustive search.

Our account meshes well with previous theories of simultanagnosia. Several authors have previously noted that even though simultanagnosic patients may have normal eye-movement patterns in scanning (Rizzo & Hurtig, 1987), they fail to completely explore a complex image and have a tendency to report the same object more than once (Coslett & Saffran, 1991; Hécaen & Ajuriaguerra, 1954; Luria, 1959). On the basis of a detailed study of a single case, Coslett and Saffran (1991) have suggested that the core deficit is an "impairment in the integration of object identity and spatial location information" (p. 1542). They showed that their patient had normal object and word recognition, and intact processing of purely spatial information, for instance in reaching. Thus, the "what" and the "where" systems of visual information processing were preserved, but they could not be properly linked or bound together.

This theory fits well to our patients, most of whom had intact feature and object recognition as well as adequate spatial orienting and grasping. The disconnection of identity information from spatial information implies that when the counted items were rectangles of the same size, shape, and color, nothing was left to help the patients discriminate which items they had already counted. This hypothesis also predicts that differentiation of the counted items along a nonspatial dimension, for instance color, should improve counting. This prediction was verified in S.T.E., whose error rate on sets of 3 or 4 items dropped from 34.1% to 2.1% when the rectangles were presented in different colors. Another patient, G.O.U., did not benefit much from such color differentiation, but he might have suffered from an additional deficit of color processing, as attested by a lack of pop-out from color in visual search.

Models of Subitizing

Having established the nature of the patients' counting deficit, we may now turn to explanations of their excellent quantification of sets of 1, 2, and sometimes 3 items.

Fast Counting

Gallistel and Gelman (1991) proposed that even small sets of items are quantified by serial counting, albeit with faster speed than for larger sets. This view seems incompatible with the present data. Sets of 1 items are special because they do not require any memory of previously counted items. However, even with sets of 2 or 3 items, it becomes crucial to keep track of previously counted items. If our patients had used counting with sets of 2 or 3 items, their inability to individuate the targets on the basis of spatial location should have yielded some errors of underestimation or overestimation. The absence of any quantification errors with 2 and sometimes 3 items in patients with severe deficits of serial visual exploration suggests that these sets were quantified using a parallel procedure (subitizing) rather than a serial one.

Canonical Patterns

Mandler and Shebo (1982) have attributed the fast quantification of small numerosities to a process akin to object recognition: Subjects would recognize the characteristic geometric configuration of sets of 1, 2, or 3 objects and learn their numerical labels (e.g., a triangle = 3). Pattern recognition would fail for sets of 4 or 5 items, at which point the subject would then resort to slow counting. Contrary to the prediction of this model, we found no consistent relationship between subitizing and pattern recognition. Only one patient (I.S.A.) showed a fully intact recognition of canonical patterns of dots similar to those found on a dice. In all other patients, quantification of patterns as simple as an equilateral triangle was far from perfect when the displays were presented for 200 ms, yet they remained able to identify sets of 1 and 2 items with near-perfect accuracy at this exposure duration. This dissociation suggests that small sets were probably not quantified by a pattern recognition process. That subitizing is found even with linear arrays (e.g., Atkinson, Francis, & Campbell, 1976) also argues against the canonical pattern account (for discussion, see Dehaene, 1992; Trick & Pylyshyn, 1994).

In passing, it is worth noting that outside their subitizing range, all simultanagnosic patients performed better with canonical patterns than with random patterns. The two patients tested with an unlimited presentation of canonical patterns (G.O.U. and S.T.E.) achieved almost perfect quantification. In three patients (S.T.E., L.E.F., and I.S.A.), RTs and verbal reports suggested that canonical patterns were not counted. The relative preservation of canonical pattern recognition in simultanagnosic patients is somewhat paradoxical because the identity of these patterns was defined only by the relative locations of otherwise identical and unconnected rectangles. If simultanagnosia is characterized by an impaired processing of object locations, perhaps due to an inability to coordinate identity information with spatial information, why was relative object location still available for the recognition of familiar patterns?

This paradox is not specific to the present study, but has been lingering in many previous cases of simultanagnosia. Luria's (1959) patient, for instance, could readily recognize and name a pattern of six dots arranged in a rectangle, even though he was unable to count the dots. Luria noted that "there is an apparent paradox is the fact that the patient can perceive a unified structure while being at the same time unable to appreciate a complex of unrelated elements" (pp. 445–446). Coslett and Saffran (1991) also outlined what they viewed as

> perhaps the most puzzling aspect of the performance of patients with simultanagnosia: when confronted with a complex visual array these patients often report 'seeing' only a single item, yet with confronted with a large depiction of a single 'object' [. . .] such as a drawing of an elephant, subjects generally report seeing the entire object rather than [its] components (e.g. an ear, trunk, etc.) (p. 1541).

This dissociation was demonstrated in a well-controlled situation by Levine and Calvanio (1978): Their patients could identify most letters in a three-letter word, but often reported only one letter from a three-letter nonword.

As a tentative resolution of this paradox, Coslett and Saffran (1991) proposed that "stored structural information about the visual form of [a single object] facilitates the perception of [its] component parts" (p. 1541). This hypothesis, however, may work only if the "stored structural information" encompasses a description of the spatial location of the component parts (e.g., the relative placement of letters in the word "CAT," or of the dots forming an equilateral triangle). The relative preservation, in simultanagnosia, of the spatial relations between the parts of a familiar object, but not between the objects in an unfamiliar scene, remains to be satisfactorily explored and explained.

Estimation and Object Normalization Models of Subitizing

Human subjects can estimate the numerosity of displays of several hundred items, and the precision of their estimation decreases as numerosity increases (e.g., Krueger, 1982; Mandler & Shebo 1982; Taves, 1941). Some authors have suggested that estimation processes, when applied to small numerosities, become sufficiently precise as to allow for an exact quantification of the display (Averbach, 1963; van Oeffelen & Vos, 1982). Thus, subitizing would not be a distinct process, but just a form of "precise estimation." Several computational models of numerosity estimation are available, all of which are based on an estimation of the density of objects and of the visual area that they occupy (Allik & Tuulmets, 1991; van Oeffelen & Vos, 1982; Vos et al., 1988).

A related "object normalization" model has been proposed by Dehaene and Changeux (1993). In their simulation of a simplified neuronal network, input objects are simulated as distributions of activation of various sizes over a one-dimensional input retina. A first network locates the input objects and allocates to each object, regardless of its initial size, an approximately constant pool of active units (object normalization stage). Numerosity detection units then sum these normalized activation to get an estimate highly correlated with numerosity. Because numerosity is estimated as a sum of random variables, accuracy decreases with the number of terms in the sum. The models fails to reliably discriminate 4 from 3 or 5, and it is assumed that human adults switch to counting at this point. Beyond the specifics of the simulation, Dehaene and Changeux's (1993) model rests on the idea that numerosity estimation is parasitic on more general routines of object isolation and separation from the background, that work in parallel across the visual display.

How can a model that does not incorporate a sharp subitizing limit explain the sharpness of the dissociations observed here between perfect quantification of n items and very impaired quantification of $n + 1$ items? According to both estimation and object normalization models, subjects have learned to stop relying on approximate evaluation and start counting at the point at which evaluation becomes too noisy (e.g., between 3 and 4). Both models may therefore account for the present data by supposing that even though their counting procedure was impaired, the patients still switched to counting as soon as their evaluation became imprecise. This account predicts that if subjects had been given feedback as to how bad their counting was, and if they had been taught to rely on evaluation rather than on counting, their quantification performance would have improved. This prediction remains to be tested.

FINST Model

Trick and Pylyshyn (1993, 1994) attributed subitizing to the parallel assignation of pointers called FINSTs (for FINgers of INSTantiation) to each object in a visual display. It is assumed that FINSTs are available in a limited number (e.g., 4), and that subjects can rapidly and preattentively determine how many FINSTs have been assigned. This explains both the limited capacity and the speed of subitizing. The FINST hypothesis is not as ad hoc as it may first seem; the existence of a limited number of FINSTs was inferred independently from multiple object tracking experiments (Pylyshyn & Storm, 1988), as well as from theoretical considerations on the computational requirements of visual routines (Ullman, 1984).

The FINST model fits well with the present data, because it predicts that subitizing is parallel and is based primarily on preattentive processing, and may therefore dissociate from serial counting. One potential difficulty is the small size of the subitizing range observed in our patients (up to 2 or occasionally 3 items). This might be accounted for, however, by a postlesional reduction in the number of FINSTs. The main difficulty for testing the FINST model, at present, is that the mechanisms of pointer assignment and the variables affecting the number of available FINSTs remain largely unspecified.

Conclusion

The neuropsychological dissociations reported here indicate that quantification of small sets is not based on serial counting and presumably relies on spatially parallel processing (subitizing). Even if the exact nature of subitizing remains unknown, some alternatives have been rejected, most notably the notion that subitizing is based on the recognition of invariant geometric configurations. We would like to stress, in conclusion, that subitizing may not necessarily be based on a single procedure. With short presentation, one patient (S.T.E.) made errors even when 1 or 2 objects were presented. Three other patients (A.M.E., G.O.U., and L.E.F.) made few errors with 2, but a very large number of errors with 3 items. Finally, for another patient (I.S.A.) the discontinuity was clearly between 3 and 4 items. This variability in the preserved "subitizing range" might reflect prelesional individual differences as well as a variable postlesional reduction in speed of processing or amount of processing resources. It seems equally possible, however, that the fast quantification of sets of 3 items might be based on procedures different from those used with sets of 1 or 2 items.

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Appendix

The Consequences of a Low-Level Visual Deficit on Counting Performance

Let us assume that a set of n items is presented visually, and that each item has a probability p of being missed, for instance, because it falls in a degraded region of the visual field. Then the expected error (E) rate in counting, which is the probability of missing at least one item, is given by $E(n) = 1 - (1 - p)^n$. The average response (R), which is the expected value of the number of items that are not missed, is given by $R(n) = n(1 - p)$. Finally, the average absolute error (A), which is the expected value of the number of missed items, is given by $A(n) = n\,p$.

Figure A1 shows the evolution of E, R, and A when n varies from 1 to 6 items, for different values of the parameter p. Obviously, visual field deficits can never yield a sharp discontinuity as observed in our patients, with good performance on some numerosity n and a sudden onset of errors to numerosity $n + 1$. Rather, the equations imply that the error rate is always a decelerating function of n, with $E(1) > 0$ for $p > 0$ (i.e., the error rate on a single item should never be zero). Likewise the average response and the average absolute error are linearly increasing without any discontinuity. Comparison with actual data, for instance those of Patient I.S.A. (see Figure 5), show that even though the model fails to account for the preservation of subitizing, it fits well with the subjects' counting errors. Most patients indeed appeared to forget, more or less randomly, to count some of the targets.

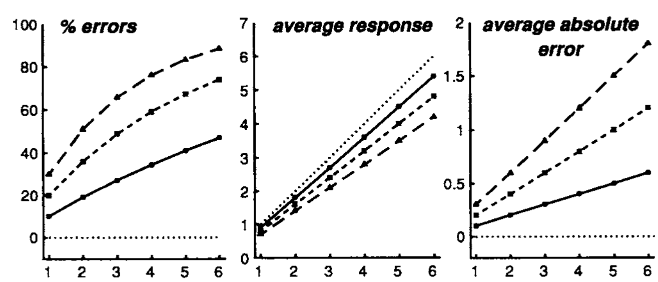

Figure A1. Quantification performance predicted by a counting model in which each item has a fixed probability p of being missed (the three curves correspond to p = .1, .2, or .3). This model cannot account for the discontinuities observed in real patients' performance.

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