11 Numerical and Spatial Intuitions: A Role for Posterior Parietal Cortex?

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Historically, many mathematical advances have been developed through the use of conceptual mappings between numbers and space. From the most elementary aspects of mathematics, such as the notion of measurement, all the way up to the concepts of the real number line, Cartesian coordinates, the complex plane, and even the proof of Fermat's Last Theorem, metaphors by which numbers are made to correspond to spatial positions permeate mathematical thinking (Dehaene 1997; Singh 1997). The evolution of these culturally defined representations of number has been critical to the development of mathematics. In this chapter, we review and update our previous models (Dehaene et al. 2003; Hubbard et al. 2005) discussing the neural mechanisms that might underpin these cultural achievements. We begin by reviewing recent behavioral, patient, and transcranial magnetic stimulation (TMS) data showing that certain aspects of numerical understanding depend on spatial representations (for reviews of the behavioral literature, see also Fias and Fischer 2005; Gevers and Lammertyn 2005). We then turn to neuroimaging data in humans that suggest how the deep connection between numbers and space may be mediated by circuitry in the parietal lobe. Drawing on recent work in monkey physiology and human neuroimaging studies establishing tentative homologies, we then present a refined hypothesis concerning specific neural regions in the intraparietal sulcus (IPS) involved in these numerical and spatial processes, including the human homologues of the lateral intraparietal (hLIP) and ventral intraparietal (hVIP) regions. To date, these two lines of research have been largely independent, as most studies of numerical cognition have been conducted in humans using functional imaging, while the most detailed studies of spatial processing have been conducted in monkeys, using single-unit electrophysiology. However, this division is breaking down, as single-unit data have revealed "number neurons" in the macaque IPS, while many recent human neuroimaging studies have focused on establishing human-monkey homologies in the parietal lobe. We conclude by discussing the development of numerical-spatial interactions within the context of the "neuronal recycling" hypothesis (Dehaene 2005; Dehaene and Cohen 2007).

Behavioral Studies of Numerical Spatial Interactions

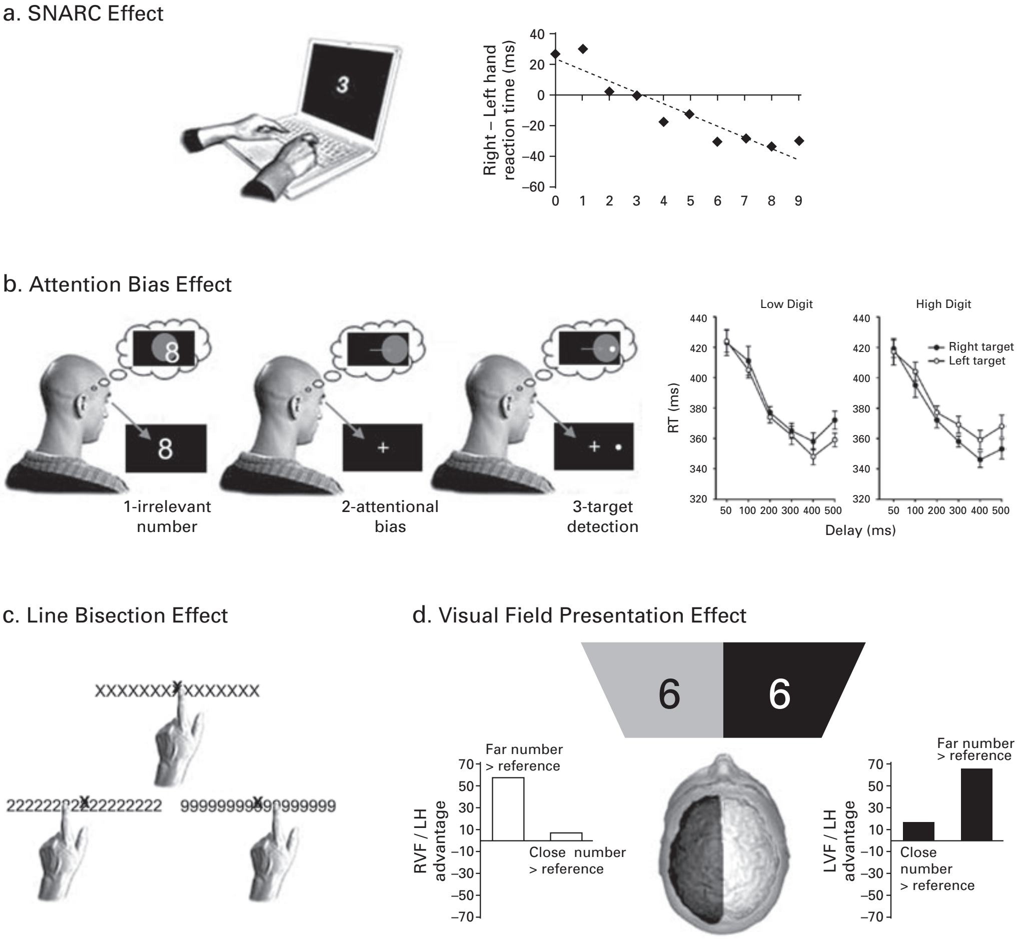
Numerous behavioral paradigms have demonstrated a close connection between numbers and space, in which smaller numbers are represented on the left side of space, and larger numbers on the right. In this section, we will examine three important questions that have guided research in this area. First, how automatic is this association between numbers and space? Second, what level of spatial representation is involved? And third, what role do cultural factors play in the orientation of these numerical-spatial associations?

Automaticity of Numerical-Spatial Interactions

The simplest demonstration of a connection between numbers and space is the spatial-numerical association of response codes (SNARC) effect (Dehaene et al. 1993). When subjects are asked to classify numbers as even or odd (parity judgment), smaller numbers are responded to more quickly when responses are made on the left side of space, while larger numbers are responded to more quickly when responses are made on the right (figure 11.1a).

This association of numbers and space occurs despite the fact that the task itself has nothing to do with numerical magnitude. Indeed, the SNARC effect can occur with non-numerical tasks such as judging phonemic content of number words (Fias et al. 1996) or even in tasks where the digit itself is completely irrelevant to the task. In one series of experiments, subjects were asked to perform an orientation discrimination task on a triangle or line superimposed on a digit, and to respond with the left or right hand. In this task, a SNARC effect was observed, suggesting that numerical magnitude was processed automatically (Fias et al. 2001; Lammertyn et al. 2002). However, this effect was reduced or absent when subjects were asked to report the colors of the digit, or when asked to identify a shape (circle or square) superimposed on the digits.

Even simply presenting a digit automatically draws attention to either the left or right visual field based on the relative size of the number (Fischer et al. 2003). Fisher and colleagues presented single-digit numbers (1, 2, 8, or 9) at fixation, followed by a target in either the left or right visual field that participants responded to as quickly as they could (detection reaction time [RT]). The magnitude of the number influenced the direction of the allocation of attention, and thus the detection RT (figure 11.1b). Digits 1 and 2 automatically directed attention to the left visual field and thus facilitated the response to left-sided targets, whereas the opposite was true for 8 and 9, even though the digit was noninformative and completely irrelevant. In a recent follow-up it has been shown that these shifts of attention can have perceptual consequences. For instance, numerical cues induced the phenomenon of prior entry, in which objects at attended locations are perceived as appearing earlier than objects at nonattended locations (Casarotti et al. 2007). Similarly, in a backward priming experiment, Stoianov and colleagues (2007) found that



a. SNARC Effect

b. Attention Bias Effect

c. Line Bisection Effect

d. Visual Field Presentation Effect

Figure 11.1
Behavioral studies demonstrating numerical-spatial interactions. (a) SNARC effect. Subjects respond whether a number is even or odd. Right-minus left-hand reaction time differences are plotted, with values greater than 0 indicating a left-hand advantage. Adapted from Dehaene et al. (1993). (b) Attention bias effect. Presentation of a noninformative digit at fixation leads to an automatic shift of attention to the left or right, and subsequently faster responses to visual targets. Graphs indicate reaction times to detect a visual target on the left or right side of space after presentation of a "low" or "high" digit. Open symbols indicate left-sided targets and filled symbols, right-sided targets. Adapted from Fischer et al. (2003). (c) Line bisection effect. When asked to point toward the midpoint of a line, subjects are accurate when the line is composed of x's (center indicated by bold x). However, when the line is composed of 2's or 9's, pointing deviates from the midpoint. (d) Visual field presentation effect. When a number is presented in one visual field, an interaction between numerical distance and visual field is observed. Numbers that are smaller than the standard show an advantage for LVF/RH presentation, while numbers that are larger than the standard show an advantage for RVF/LH presentation. Adapted from Lavidor et al. (2004).

responses for smaller numbers were faster when they were followed by a cue on the left side of the screen than when they were followed by a cue on the right side of the screen; the converse was true for larger numbers. This backward priming effect suggests that it takes a brief amount of time for the processing of numerical magnitude to evoke a spatial location. However, other recent studies have demonstrated that the presence of such orienting of attention is not entirely automatic, but rather is sensitive to top-down control and task set (Galfano et al. 2006; Ristic et al. 2006).

In a third demonstration of automatic numerical-spatial interactions, line bisection can be biased when the lines are composed of numbers (Calabria and Rossetti 2005; Fischer 2001). When asked to indicate the midpoint of a line composed of x's, subjects were accurate. However, when asked to indicate the midpoint of a line composed of either the digit 9 or the French word *neuf* (nine) subjects deviated to the right. When the line is composed of 2's or the French word *deux* (two) subjects deviated to the left (figure 11.1c). The suggestion is that the numbers automatically bias attention to the left or the right, and that the bisection of the lines therefore deviates in the same direction.

Spatial Reference Frames

A second question relevant to our purposes here is to determine the coordinate frame in which the SNARC effect arises. Several findings have suggested that an abstract, effector-independent cross-modal representation of space is involved. For example, it is known that the SNARC effect occurs even when the hands are crossed: large numbers continue to be associated with the right-hand side of space, even when responses on that side are made with the left hand (Dehaene et al. 1993). This observation suggests that the effect depends on eye- or world-centered coordinates, rather than hand-centered coordinates (although hand also makes an independent contribution; see Wood et al. 2006). Similar data from cross-modal visual-tactile attentional studies have shown that noninformative tactile stimuli to either hand improved detection thresholds on the same side of space even when the hands are crossed (Spence et al. 2000), suggesting that similar mechanisms may underlie both the spatial representation in the SNARC effect and in cross-modal spatial cuing. Neuroimaging studies of these cross-modal cuing effects consistently find parietal lobe activation (Kennett et al. 2001; Macaluso and Driver 2005; Macaluso et al. 2003), a point we will return to.

Additionally, the SNARC effect arises when subjects are asked to perform the parity judgment by pointing (Fischer 2003) or by moving their eyes, instead of a manual response (Fischer et al. 2004; Schwarz and Keus 2004). Finally, it has recently been shown that it is possible to obtain a SNARC effect with foot-pedal responses, demonstrating that the effect is not merely linked to effectors involved with writing but is a more general stimulus-response compatibility effect (Schwarz and Müller 2006). Bearing in mind that a noninformative digit automatically biases attention toward the left or right (Fischer et al. 2003), even though the response has nothing to do with the digit, these results suggest that numerical-spatial interactions occur in effector-independent, stable spatial coordinate frames.

A related question is the relation between the SNARC effect and the Simon effect, in which responses are faster when the stimulus and response occur at corresponding spatial locations. In the case of the SNARC, relative numerical magnitude may evoke corresponding spatial locations in representational space rather than physical space. Using the additive-factors method (AFM), two studies have yielded contradictory results. Mapelli and colleagues (2003) found that the SNARC effect, unlike the Simon effect, did not decay with time, and that it did not interact with the Simon effect. On this basis, they argued that the two effects were distinct. However, Keus and colleagues (2005), using the same AFM logic, found that the two effects did interact, suggesting that they share a common stage. More recently, Gevers and colleagues (2005) noted that both the SNARC and the Simon effect violate one of the assumptions of the AFM, namely, stage robustness, and as such the AFM logic is not appropriate for these questions. Rather, they showed that whether the SNARC and Simon interacted depended on the task relevance of the magnitude code (parity judgment vs. magnitude comparison), thereby demonstrating that the two effects do not conform to the AFM logic. To account for their results, Gevers and colleagues proposed a "dual-route" model of the SNARC, which involves activation of spatial codes indirectly via numerical codes, and which predicts a slight delay between stimulus onset and the elicitation of the spatial code, as seen in the Stoianov et al. (2007) study mentioned earlier. This account also suggests a partially shared architecture for the SNARC and Simon effects (see also Rusconi et al. 2006).

Another related question is when do these numerical-spatial interactions arise in the processing chain leading from stimulus to response? A recent study using a dual-task paradigm demonstrated a backward compatibility effect by showing that when subjects were asked to verbally respond "one" or "two" for different stimuli, even though digits were not presented, the automatic activation of numerical information interfered with responding to the orientation of an arrow (Caessens et al. 2004). This study indicates that SNARC-like influences occur at a task- and modality-independent level. Other studies have suggested that the SNARC effect best correlates with the response-locked (as opposed to stimulus-locked) event-related potentials (ERPs) and begins to emerge at a response selection stage (Gevers et al. 2006; Keus et al. 2005). However, given the delay in the elicitation of the spatial code seen in previous studies, and the relatively short response times in a traditional parity judgment task (400 to 500 ms), this temporal overlap may obscure other ERP components, such as those linked with shifts of attention, that may also play an important role in the genesis of the SNARC effect. These methodological considerations suggest that it may be premature to conclude that the SNARC effect is elicited only after substantial processing.

Indeed, interference between numerical and spatial information can arise even from spatial congruity of the stimulus, rather than the response (figure 11.1d; see Lavidor et al.

2004). The classic "numerical distance effect" is the finding that responses are increasingly faster as the numerical distance between the compared numbers increases (Dehaene et al. 1990; Moyer and Landauer 1967). However, when numbers were presented to the left (LVF) or right (RVF) of fixation, the magnitude of the distance effect was modulated, such that numbers smaller than the standard showed an advantage for LVF presentation, and numbers that are larger than the standard showed an advantage for RVF presentation. This effect is highly reminiscent of the SNARC effect (compare figures 11.1a and 11.1d). Taken together, these results suggest that numerical-spatial interactions arise at a central level, independent of input-modality or output-effector, and that they depend on spatial compatibility in both the input and output processes.

Cultural Factors

Even though these associations are automatic and depend on abstract representations of number and space, the *direction* of the effect—smaller numbers left, larger numbers right—might be determined by cultural factors such as the orientation of writing or the conventional orientation of mathematical graph axes. For example, American children do not show a SNARC effect until age nine, showing that substantial education is required before these links become automatic (Berch et al. 1999). Indeed, the SNARC effect tends to reverse in Iranian subjects who write from right to left (Dehaene et al. 1993; Zebian 2005). As Fias and Fischer (2005) note, the direction of reading influences a whole host of ordering behaviors, and its influence is probably not limited to the SNARC effect.

Interestingly, when children are asked to map numbers onto a spatially oriented line, their responses change with age from a logarithmic to a linear encoding (Siegler and Opfer 2003) between the ages of seven and nine. However, this change seems to occur in stages, as seven-year-old children are likely to map the range 0 to 100 in a linear fashion but map the range 0 to 1000 in a logarithmic fashion. That is, they dedicate more space to small numbers than to large numbers, placing 10 near the middle of the 0 to 100 segment, rather spacing the numbers equally across the entire range. More recently, Opfer and Siegler (2007) replicated this developmental trend, and have shown that training on just one number (5, 150, or 750) can lead to rapid recalibration from logarithmic to linear representations in eight-year-old children who demonstrated logarithmic scaling of the mental number line on a pretest. This feedback was most effective where the discrepancy between the linear and logarithmic representations was greatest (at 150) and generalized across the entire mental number line in an all-or-none fashion, even though only one value was trained.

In the Amazon, Australia, and Africa, one can still find some cultures with a drastically reduced verbal lexicon for numbers. These cultures provide a more extreme situation for studying cultural universals and cultural differences in the number domain. Gordon (2004) studied the Piraha, who only have names for one and two, while Pica and colleagues (2004) studied the Munduruku, who have names for numbers about up to five. In both cases, a competence for approximate numerosity was demonstrated, suggesting that this intuition arises in a strictly nonverbal form even in remote cultures without formal education. For instance, adult and children Munduruku could perform an approximate addition task where one set of dots was added to another set of dots in a can, and the task was to decide if the total was larger or smaller than a third number. Even with very small numbers, in a subtraction condition where Western control subjects could perform with exact precision (e. g., 6-4), the Munduruku performance remained approximate and could be modeled mathematically by Weber's Law, suggesting that their spontaneous representation of number is an approximate logarithmic number line. Recently, we were able to show that uneducated Munduruku adults also have intuitions of number-space mappings (Dehaene et al. 2008). When presented with a nonsymbolic version of the Opfer-Siegler task (Siegler and Opfer 2003), with a horizontal line labeled with one dot at left and ten dots at right, they spontaneously understood that other numbers go to specific places on this physical number line. Furthermore, like young children with larger numbers, they spontaneously adopted a logarithmic spacing: for them the middle of the interval 1-10 was closer to the geometric mean (3 or 4) than to the arithmetic mean (5.5). It is likely that experience with counting, arithmetic, measurement, or other aspects modifies this internal representation by giving us access to a linear coordinate scheme, but exactly which cultural factors are involved and whether they also affect the direction of the SNARC effect remains unknown.

Studies of cultural influences on the SNARC effect are made more difficult because mathematical conventions are now essentially universal and often conflict with other cultural conventions. For instance, Japanese subjects were faster to respond to small numbers with the lower response button and large numbers with the upper response button (Ito and Hatta 2004), despite the fact that Japanese subjects use both left-to-right (like Western subjects) and top-to-bottom (which would have predicted the opposite pattern of SNARC effects) writing systems. It is possible that this discrepancy is due to graphing conventions (where small = bottom left). In another recent study, Chinese speakers in Taiwan were tested with three different writing systems, which are used in different writing situations. Arabic numerals appear in horizontal text, whereas simple Chinese characters appear in vertical text. Complex Chinese characters are used only in formal situations, such as check writing, and are not associated with a particular writing direction. A horizontal, left-to-right SNARC effect was found for the Arabic numerals, but not for either of the other two systems, while a vertical top-to-bottom SNARC was found for the simple Chinese characters, but not for the other two systems (Hung et al. 2008). These data add weight to the idea that the orientation of the SNARC effect is influenced by the direction of writing and demonstrates that the mappings are flexible, depending on numerical context.

Additionally, priming different types of spatial representation affects the orientation of the SNARC effect (Bachtold et al. 1998). Subjects were presented with a magnitude task (greater or less than 6) after being primed with either an image of a ruler or an image of a clock. After being primed with a ruler, the standard SNARC effect was observed (small-left, large-right). However, after being primed with a clock face, subjects showed a reverse SNARC effect (small-right, large-left) consistent with the representation of time on the clock face.

In sum, various paradigms suggest that numbers automatically elicit task-, modality-, and effector-independent spatial representations, even when these spatial representations are not strictly relevant to the task. Although cognitive and cultural factors clearly play some role in the orientation of these effects, the existence of spatial-numerical interference is robust. In the next section we relate these effects to monkey physiology and human neuroimaging studies of parietal regions involved in the appropriate representations of numbers and space.

Patient and TMS Studies Examining Numerical-Spatial Interactions

Joint deficits of space and number are frequently observed in patients with lesions of the parietal lobes. Classic evidence for this comes from studies of patients with Gerstmann's syndrome, which often involves dyscalculia, and spatial problems such as left-right confusion and finger agnosia (Benton 1992; Gerstmann 1940; Mayer et al. 1999; Roux et al. 2003). Recently, a case of pure Gerstmann's syndrome due to a subangular lesion has been identified (Mayer et al. 1999). After substantial testing of all the elements of Gerstmann's syndrome, the authors suggested that the common deficit linking the symptoms in this patient was a deficit in visuospatial manipulations, consistent with our hypothesis of numerical-spatial interaction in the parietal lobe. Interpretation of such symptom-association data remains complicated, however, because it could be due to the mere anatomical proximity of functionally distinct systems. Indeed, numerous studies have questioned the unity of Gerstmann's syndrome by showing that its defining features can be dissociated in both patient (Benton 1992) and intracranial stimulation studies (Roux et al. 2003).

Recent studies support a role for the right parietal lobe in the connection between numbers and space by demonstrating distortions in number processing in patients with hemi-spatial neglect (Vuilleumier et al. 2004; Zorzi et al. 2002). Patients with neglect ignore the contralesional (usually left) portion of space, including internal representational space in mental images, a condition known as "representational neglect" (Bisiach and Luzzatti 1978; see figure 11.2a).

In a classic test of neglect, patients, when asked to bisect a line, neglect the left half of the line, and therefore place the perceived midpoint of the line to the right of center (Driver and Vuilleumier 2001). In one recent study Zorzi and colleagues (2002) demonstrated that neglect patients have deficits in numerical tasks that closely correspond to those seen in physical line bisection tasks. When patients with neglect were asked to state the midpoint number of various numerical intervals—say, to give the numerical midpoint of 3 and 15—they deviated "to the right" (toward larger values), and for the smallest interval (3) they deviated "to the left" (toward smaller values), consistent with the "cross-over" effect

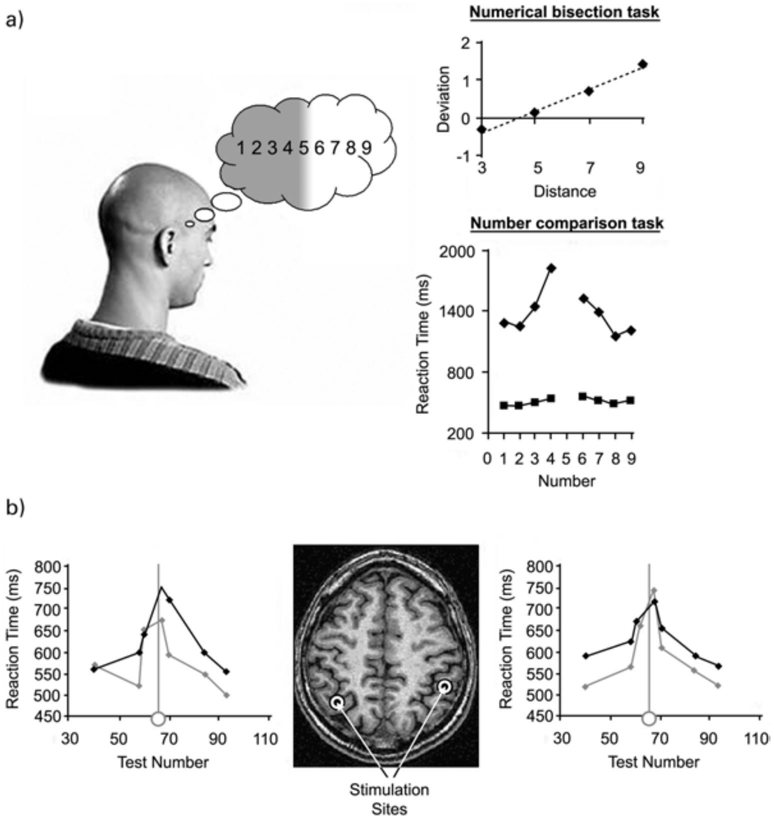


Figure 11.2
Hemispheric effects in numerical-spatial interactions. (a) Neglect patients also demonstrate severe deficits in numerical distance and number bisection tasks. The upper graph shows the deviation on a number-interval bisection task, as a function of interval size (adapted from Zorzi et al. 2002), while the lower graph shows reaction times on a magnitude judgment task with 5 as the standard (adapted from Vuilleumier et al. 2004). (b) When rTMS is applied to the angular gyrus, responding to a number greater than the standard takes longer than in the no-stimulation condition. Adapted from Göbel et al. (2001).

observed in patients with spatial neglect. This was despite the fact that both the problem input and the response were given in a nonspatial spoken form. This numerical bias reflects a purely representational form of neglect (Bisiach and Luzzatti 1978), and suggests that numerical bisection involves an internal stage of representation on a spatially oriented "number line." Patients with right parietal lobe damage but no neglect do not show this pattern (Zorzi et al. 2002; for data suggesting that line and number bisection are doubly dissociable, see Doricchi et al. 2005). Additional follow-up studies have shown that the effect of neglect differs across tasks, such that the SNARC effect remains unaffected, despite impaired bisection performance, suggesting that the effect of neglect may vary depending on whether the task requires implicit or explicit access to the mental number line (Priftis et al. 2006). A second study showed that these representational deficits extend to the clock and ruler tasks described above (Vuilleumier et al. 2004; figure 11.2b). Wearing leftward adapting prisms tends to improve both spatial (Frassinetti et al. 2002; Rossetti et al. 1998) and representational neglect (Rode et al. 2001), including numerical neglect (Rossetti et al. 2004), further suggesting that the neural mechanisms that underlie spatial abilities are critical for certain numerical tasks.

Joint deficits of space and number can also be induced by TMS in normal subjects. TMS over the left angular gyrus, but not the left supramarginal gyrus or corresponding sites in the right hemisphere, both disrupted performance on a visuospatial search task and caused a deficit in numerical processing when subjects performed a magnitude judgment, but only for numbers greater than the midpoint of the interval (Göbel et al. 2001; figure 11.2c). In a follow-up study, Göbel and colleagues (2006) compared right parietal TMS with occipital stimulation while subjects were asked to report the midpoint of a verbally presented numerical interval. Parietal stimulation led to neglect-like responses, replicating the results found in patients with neglect, whereas occipital stimulation had no effect.

In the first study to directly examine the effects of TMS on the SNARC effect, Rusconi and colleagues (2008) tested both the SNARC and the Simon effect while they administered TMS over one of four different sites: anterior or posterior portions of the posterior parietal lobule (PPL) in either the left or right hemisphere. They found that stimulation over anterior PPL sites interfered with the Simon, but not SNARC, effect, whereas stimulation of posterior PPL sites interfered with both the Simon and the SNARC effect, consistent with the behavioral evidence reviewed earlier, which suggests that the two effects depend on partially shared neural circuits. Note that this localization is consistent with the role we had previously proposed for posterior parietal regions, such as the human homologue of lateral intraparietal (hLIP; see more in the next section), in the generation of the SNARC effect (Hubbard et al. 2005).

In general, these results suggest that numerical manipulations are critically dependent on intact spatial representations, and that the neural mechanisms of numerical-spatial interactions might be the same ones that subserve spatial cognition in the intact brain. One caveat is that both lesion and TMS effects probably encompass large amounts of cortex and thus may cause multiple independent impairments, suggesting the need for more fine-grained analysis of the neural substrates of these functions.

The Parietal Basis of Number Processing

The past ten years has seen an explosion of interest in the neural basis of basic mathematical processes such as subitizing, numerosity estimation, addition, subtraction, and multiplication (Dehaene 1997; Dehaene et al. 2004). One of the main findings from this line of research is that the neural circuitry critical for abstract representations of quantity is housed in the parietal lobe, in regions overlapping with neural circuitry involved in spatial representations.

The triple-code model of number processing (Dehaene 1992) proposes that numbers can be mentally represented in a nonverbal *quantity* representation (a semantic representation of the size and distance relations between numbers, which may be category-specific), a *verbal* system (where numerals are represented lexically, phonologically, and syntactically, much like any other type of word), and a *visual* system (in which numbers can be encoded as strings of Arabic numerals). The quantity system is thought to be located in the parietal cortex, and this system may be critical for mediating the observed interactions between numerical and spatial representations.

Functional magnetic resonance imaging (fMRI) has been used to test this model and to localize the nonverbal quantity system. Numerous results indicate that number comparison typically involves the left and right parietal lobes. In some experiments, for instance, subjects were asked to compare two numbers and decide which one was larger (Pinel et al. 2001, 2004). Irrespective of whether the numbers were presented as digits or as words, an identical behavioral distance effect was observed. FMRI indicated that the activation of the left and right intraparietal sulci (IPS) showed a tight correlation with the behavioral distance effect: the activation signal in this region also showed an inverse relation to the distance between the numbers to be compared. On the basis of this and other fMRI experiments of arithmetic tasks such as comparison, calculation (Chochon et al. 1999), approximation (Dehaene et al. 1999), or even the mere detection of digits (Eger et al. 2003), a meta-analysis has suggested that the bilateral horizontal segment of the IPS (HIPS) may play a particular role in quantity representation (Dehaene et al. 2003). In some cases, the activation also extended to dorsal parietal sites thought to be involved in spatial attention orienting.

Crucially, the quantity system in the parietal lobe might be part of a broader network of areas involved in nonnumerical magnitude representation (Fias et al. 2003; Pinel et al. 2004). Pinel and colleagues (2004) measured fMRI responses during three tasks: luminance comparison, size comparison, and numerical magnitude comparison. Because all three tasks demonstrate a distance effect, it was possible to match task difficultly by varying the discriminability of the stimuli for each subject. FMRI revealed a network of areas that were activated during each of the three tasks. An anterior region of the IPS was activated by all three tasks, but other mid-IPS regions were activated only by numerical comparison, suggesting a distributed, partially overlapping network of regions.

To further examine the neural basis of this quantity system, Simon and colleagues (2002) used fMRI to examine the topographical relation of calculation-related activation to other spatial and language areas in the human parietal lobe. They found that manual tasks (grasping and pointing) activated a large overlapping region in the anterior parietal cortex, with the greatest extent of activation for grasping, which recruited an additional anterior intraparietal region bilaterally (possibly coinciding with area hAIP [human anterior intraparietal area]; see the next section). Posterior to this was a region selectively activated by calculation alone, specifically in the horizontal segment of the intraparietal sulcus (HIPS). The posterior parietal cortex was activated by all visuospatial tasks (grasping, pointing, saccades, and spatial attention), consistent with previous data (Corbetta and Shulman 2002). Finally, calculation and phoneme detection jointly activated a portion of the IPS lying underneath the left angular gyrus. Overall, these results suggest that calculation activates the fundus of the IPS in a region close to, or within, hVIP surrounded by a network of areas involved in manual, visuospatial, and verbal tasks.

Neurons Sensitive to Number

Several animal species spontaneously keep track of number (Dehaene et al. 1998; Hauser et al. 2000; Hauser et al. 2002) and can be trained to use symbolic representations of number in a variety of tasks (Boysen and Berntson 1989; Harris and Washburn 2005; Matsuzawa 1985). Additionally, it has been shown that many of the signatures of semantic numerosity processing, such as the distance effect, are present in macaque monkeys, suggesting a shared evolutionary basis for such effects (Cantlon and Brannon 2005, 2006). Physiological recordings have demonstrated that there are neurons in the parietal cortex of cats (Thompson et al. 1970) and macaques (Nieder and Miller 2004; Sawamura et al. 2002) that respond selectively to number (for a recent review, see Nieder 2005). These results suggest that there may be an evolutionary necessity to keep track of the number of objects and events in the environment, and that, at least at a rudimentary level, the ability to estimate numerosity may be present in many nonhuman animals.

Recently, Andreas Nieder and Earl Miller (Nieder et al. 2002; Nieder and Miller 2003, 2004) recorded from single neurons in awake monkeys trained to perform a visual number match-to-sample task. Many neurons were selectively tuned to a preferred numerosity; some responded preferentially to sets of one object, others to two objects, and so on up to five objects. The tuning was coarse, and became increasingly imprecise as numerosity increased. Importantly, a large proportion of these number-selective neurons were originally observed in the dorsolateral prefrontal cortex, but more recently another population of neurons with a shorter latency has been found in the parietal lobe (Nieder and Miller 2004). In a more recent study, Nieder and colleagues (2006) showed that some number-selective neurons also demonstrated motion selectivity, consistent with their localization to VIP, a plausible homolog of the human HIPS area active during many number tasks.

Piazza and colleagues (2004) used an adaptation method to investigate whether such numerosity tuning exists in humans, and thus to link human fMRI responses to those obtained with monkeys. During fMRI, they repeatedly presented participants with sets of dots with a fixed number, say, sixteen. The purpose was to "adapt" the neural population coding for this value, thus leading putative human number neurons to progressively reduce their firing rate, as observed in macaque electrophysiological experiments (Miller et al.

1991). They then presented occasional deviant numbers, which ranged from half to twice the adapted number. FMRI revealed that only two regions, the left and right IPS, responded to the change in numerosity by increasing their activation in relation to the distance between the adapted number and the deviant one, regardless of the direction of the change (more or less dots). In a follow-up study, Cantlon and colleagues (2006) replicated these findings and showed that four-year-old children demonstrated similar adaptation in the IPS, which overlapped with the regions showing adaptation in the adults. Interestingly, these effects were stronger in the right hemisphere than in the left, suggesting a potential developmental difference between the two hemispheres for the representation of numerosity. Piazza and colleagues (2007) have recently extended this adaptation effect to a cross-notation paradigm, where it was shown that digits lead to adaptation for the corresponding number of dots, and vice versa, but not when the numerosities are different, thereby showing that digits and numerosity converge on the same neural populations in adult subjects.

These human fMRI and monkey electrophysiological data yielded similar tuning profiles, suggesting that humans and macaque monkeys possess similar populations of intraparietal number-sensitive neurons. In both the single-unit recording studies and the human fMRI studies, responses closely matched predicted responses from computational models (Dehaene and Changeux 1993; Verguts and Fias 2004). Specifically, the firing rates assumed a Gaussian distribution only if plotted on a logarithmic scale. This logarithmic compression is commonly seen in human numerical tasks (Dehaene 2002), and is reflected in decreased word-frequency with numerical magnitude, and local increases for reference numerals such as 10, 20, 50, or 100 in many of the world's languages (Dehaene and Mehler 1992). Thus, even the fine-grained properties of adult numerical abilities can be predicted from the responses of neurons in the parietal cortex.

The Parietal Basis of Spatial Cognition

Recent work in both electrophysiology (Cohen and Andersen 2002; Colby and Goldberg 1999) and neuroimaging (Orban et al. 2004) has begun to converge on specific regions of the parietal lobe as the possible neural bases for the spatial representations that we discuss here. On the basis of architectonic (Lewis and Van Essen 2000a), connectivity (Felleman and Van Essen 1991; Lewis and Van Essen 2000b), and physiological criteria, the intraparietal sulcus has been divided into numerous subregions that represent space in a variety of different frames of reference. Identification of putative human homologs of macaque IPS regions is tentative, both because the parietal and frontal cortex is differentially expanded in humans compared with similar regions in macaques (Van Essen et al. 2001) and because direct comparisons between monkey and human fMRI responses to the same stimuli have revealed important differences (Orban et al. 2006; Orban et al. 2003). Nevertheless, the overall pattern of posterior-to-anterior organization, with a systematic transformation from sensory to effector-specific properties, presents striking parallels with that observed in previous studies of monkey physiology (Culham and Valyear 2006; Simon et al. 2002). We will focus on three of these putative homologies, areas hLIP, hVIP and hAIP, where *h* identifies these as putative human homologs of the aforementioned monkey areas.

Area LIP and hLIP

Many neurons in macaque area LIP are organized into a retinotopic map (Ben Hamed et al. 2001), represent target position in an eye-centered frame of reference (Colby et al. 1995; but see Mullette-Gillman et al. 2005), and are highly active during memory-guided saccades (Colby et al. 1993; Colby et al. 1996; Snyder et al. 2000). Additionally, these neurons are involved in spatial updating, even before an eye movement is made (Colby et al. 1995; Duhamel et al. 1992). Reversible inactivation of this region leads to deficits in saccade execution, demonstrating its causal role in eye movements (Li and Andersen 2001; Wardak et al. 2002).

Recent neuroimaging studies have demonstrated as many as four retinotopic maps within the posterior portion of the human intraparietal sulcus, and there is still debate as to which of these maps constitutes hLIP, and whether the additional maps are evolutionarily new (Schluppeck et al. 2005; Sereno et al. 2001; Silver et al. 2005; Swisher et al. 2007). Despite this ambiguity, recent studies have shown that posterior IPS responds in an effector-independent manner (Astafiev et al. 2003; Medendorp et al. 2005) and is jointly active for attending, pointing, and making saccades to peripheral targets (see also Simon et al. 2002). In addition, this region demonstrates delay-period activity (Schluppeck et al. 2006) and is involved in spatial updating (Medendorp et al. 2003; Merriam et al. 2003), as is macaque LIP. More recently, Morris and colleagues (2007) have used TMS to show that inactivation of this region leads to deficits in a double-step saccade paradigm. Taken together, these results suggest that at least one of the maps identified in the posterior parietal cortex is the human homolog of macaque LIP.

Area VIP and hVIP

Macaque area VIP contains populations of neurons that represent targets in either a head-centered or eye-centered frame of reference (Duhamel et al. 1997, 1998), although some receptive fields (RFs) are partially shifting or gain-modulated by eye position (Avillac et al. 2005). That is, when the eyes are moved around in the visual field, the best stimulus location either remains fixed relative to the position of the head (head-centered) or shifts partway between the position relative to the eyes and that relative to the head (partially shifting receptive fields). Additionally, many VIP neurons have joint tactile and visual motion-determined receptive fields (Duhamel et al. 1998), and are strongly driven by optic flow fields (Bremmer et al. 2002; Zhang et al. 2004). To date, two fMRI studies have attempted to identify hVIP. Bremmer and colleagues (2001) tested for regions that were conjointly activated by visual, tactile, and auditory motion. Only one such region was identified in the fundus of the IPS, anterior to hLIP, and consistent with the known organization in the monkey. In another study, Sereno and Huang (2006) mapped visual and tactile responsiveness, and demonstrated the presence of visual and tactile maps in the mid-IPS near to, but slightly mesial and superior to, the peaks of the Bremmer et al. study. They found that these maps were spatially aligned, so that voxels showing responses to a specific location in the visual field also responded to tactile stimulation on corresponding portions of the face, further suggesting that this is the human homolog of macaque VIP.

Area AIP and hAIP

Macaque area AIP represents space in hand-centered coordinates, and is crucial for fine grasping (Iwamura et al. 1994; Taira et al. 1990). Neurons in this area are bimodal (visual-tactile; see Murata et al. 2000; Saito et al. 2003; Taira et al. 1990), so that when the hand moves, the visual receptive field remains in a fixed position relative to the hand. Neurons in this area, in combination with neurons in the caudal intraparietal area (CIP), which extracts 3-D shape, are critical for correctly reaching to and grasping 3-D objects (Sakata et al. 1999; Shikata et al. 2001) and tools (Hihara et al. 2003; Iriki et al. 1996; Obayashi et al. 2001). Neurons in monkey area AIP respond in a hand-centered manner and are involved in fine grasping, but not necessarily in the transport phase of the action. Several studies have used these properties to identify hAIP (for a review see Culham et al. 2006). In the first study of this kind, regions of the IPS that responded when subjects grasped objects were identified (Binkofski et al. 1998). Interestingly, the region identified by fMRI overlapped nearly completely with a region that was damaged in a patient who demonstrated a selective impairment in fine grasping behavior (Binkofski et al. 1998). Other studies identified a region of the anterior IPS that responded more strongly to grasping than to reaching (Culham et al. 2003) or to finger pointing (Simon et al. 2002). As expected from monkey maps, activations in these regions putatively homologous to area AIP consistently lie anterior to the activations identified with the putative hLIP and hVIP.

Overlap with Numerical Activations

Crucially, the regions that have been consistently activated in arithmetic tasks overlap with, or are intermingled with, putative area hVIP, consistent with the localization derived from anatomical criteria in the monkey (see figure 11.3).

It is possible that this overlap accounts for the interaction between representations of number and space. At present, however, this co-localization remains only tentative, given that these regions have commonly been defined on the basis of average foci of brain activation in a normalized template space. Future studies should concentrate on higher-resolution studies in which hLIP, hVIP, and hAIP are identified in individual subjects. Once these regions have been identified on an individual-subject basis, activation related to number processing can be compared to these predefined regions of interest.

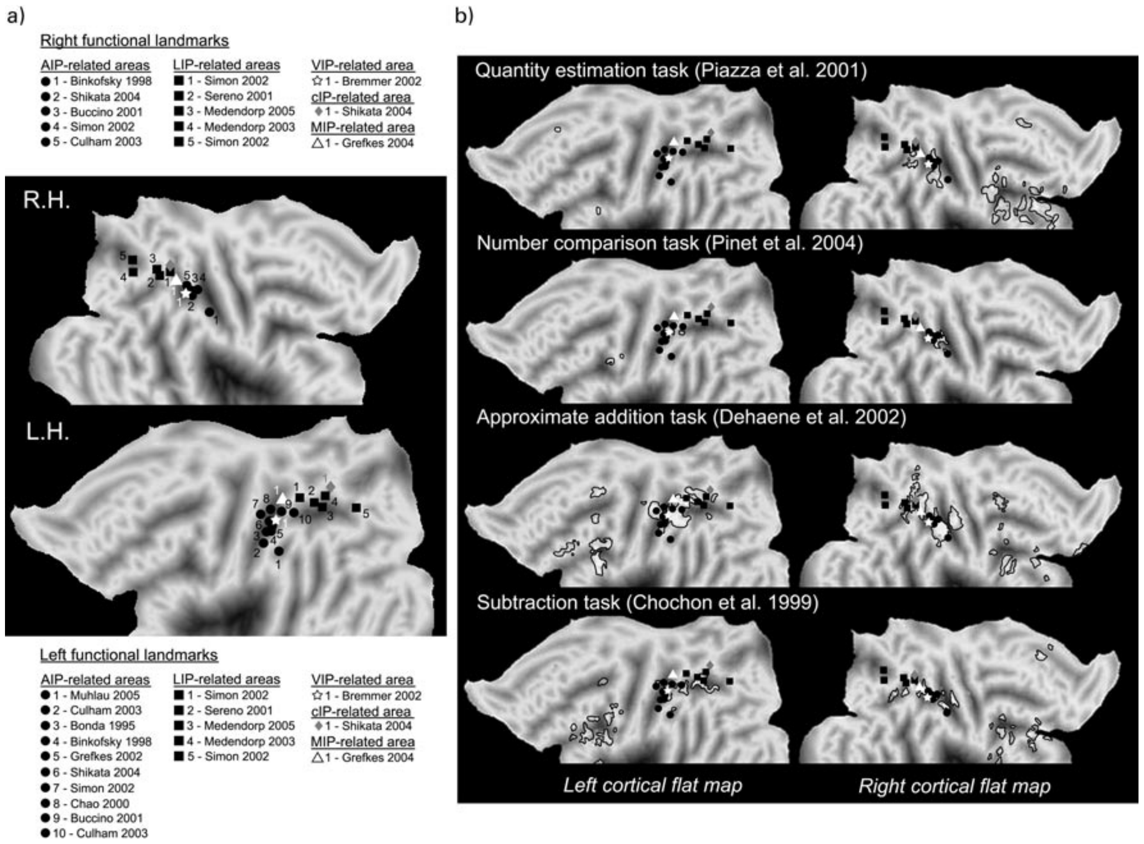


Figure 11.3
Functional overlap between numerical and spatial representations.(a) Flattened left and right hemispheres showing the peaks of numerous studies identifying human homologs of macaque regions (see legend). (b) Overlaid on these maps are the activations from four different types of numerical studies: addition, subtraction, estimation, and comparison (outlined in black). Estimation and comparison activated circumscribed regions overlapping hVIP consistent with macaque physiology data. Addition and subtraction activated larger networks, including hLIP, consistent with our hypothesis that numerical operations may also depend on hVIP-hLIP circuitry and interactions.

A Possible Role for hLIP in Shifts of Attention Along the Mental Number Line

These considerations lead us to speculate that shifts of attention along the mental number line may be mediated by shifts of attention in hLIP in the same manner that shifts of attention in the external world are mediated by hLIP. This hypothesis may explain many of the behavioral and patient data reviewed. First, the finding that the SNARC effect is present even when the hands are crossed (Dehaene et al. 1993) is consistent with the stable, eye-centered spatial representation in hLIP, and with data suggesting that multisensory (tactile-visual) attentional effects show similar remapping in space, including the activation of posterior IPS regions. Second, this hypothesis would explain why the SNARC effect is effector-independent (Dehaene et al. 1993; Fischer 2003; Schwarz and Keus 2004), given that area hLIP contains an effector-independent representation of space. Finally, this hypothesis can explain the results of the Fischer studies (Fischer 2001; Fischer et al. 2003), in which presentation of numbers leads to automatic shifts of attention to the left or to the right. We suggest that all of these effects arise from a common neural mechanism, namely, the flow of some activation from a quantity representation in area hVIP to interconnected hLIP neurons involved in programming overt and covert shifts to the contralateral side of space (Corbetta and Shulman 2002).

Similarly, in patients with neglect, we suggest that area hLIP is damaged or functionally disconnected, leading to the failure to attend to both the left side of space and the left side of the number line. It is clear that neglect is not a unitary syndrome (Halligan and Marshall 1998): some authors pin its neural substrate to the superior temporal lobe (Karnath et al. 2004), but most others place it in the parietal lobe (Mort et al. 2003). One recent proposal suggests that neglect is composed of two deficits, a spatial one, dependent on posterior superior parietal structures (including the IPS) and a memory one, dependent on the superior temporal sulcus (Malhotra et al. 2004). In light of this debate, it is interesting that transient inactivation of monkey LIP leads to neglect-like phenomena (Wardak et al. 2002; 2004). We suggest that damage to this region is responsible for not only the observed deficits in shifts of attention to external space, but also for shifts of attention along internal representations of the mental number line.

We have begun to test this idea, using fMRI, by using the classical SNARC task during whole-brain fMRI scanning (Hubbard et al. forthcoming). Subjects classified Arabic numerals as odd or even by making bimanual responses with normal or crossed hands. Four parietal regions of interest were studied, three showing lateralized activations for hands (putative hAIP), space (dorsal IPS), and saccades (putative hLIP), and a fourth active during mental arithmetic (putative hVIP). During parity judgment, number size elicited a systematic pattern of lateralized activation, which was found only in the saccade region (hLIP). In this region, a significant interaction between hemisphere and numerical size indicated that large numbers tended to cause more activation in the left hLIP, preferentially coding for the rightward side of space, while small numbers tended to cause more activation of the right hLIP, suggesting a biased attention toward the left side of space. This is the first positive evidence that this posterior parietal region, a putative homolog of macaque area LIP, may be the site of number-space interactions exemplified by the SNARC effect.

Predictions and Conclusions

Our view of the links between number and space in the parietal cortex leads to several testable predictions. First, we predict that shifts of attention along the number line make use of the same hLIP-hVIP circuitry that is involved in the development of multisensory, world-centered representations of space (Deneve and Pouget 2004; Pouget et al. 2002). This implies that the same computational transformations that support spatial updating would be critical for arithmetic operations that create shifts of the locus of activation along an internal number line (see figure 11.3). Indeed, the problem of computing a world-centered spatial representation by combining two separate population codes for eye and retinal location is formally identical to that of computing an approximate addition or subtraction by combining two population codes for numerosity (Deneve and Pouget 2004; Pouget et al. 2002). Thus, the parietal mechanisms that are thought to support spatial transformation might be ideally suited to support arithmetic transformations as well.

Future studies can test this prediction by comparing patterns of fMRI activation during spatial updating and numerical tasks. We would predict that when subjects compute additions or subtractions on numerical symbols, they will shift their attention to the left for subtraction problems, and to the right for addition problems, leading to increased activation of contralateral hLIP. Second, we predict that behavioral paradigms in which attention is shifted to the left should interfere with addition, while rightward attentional shifts should interfere with subtraction. Third, once number neurons can be recorded in animals during performance of simple addition and subtraction tasks, we predict that one should observe numerical equivalents of the partially shifting receptive fields and gain fields observed in the spatial domain.

Each of these examples might be thought of as examples of "neuronal recycling" in which preexisting neural circuits, evolved for a more basic function (in this case visuospatial processing, multisensory integration, and numerosity processing), are modified by education to perform more advanced functions (Dehaene 2005; Dehaene and Cohen 2007). Although number and space are already tightly linked by functional and anatomical links that probably exist in other animals, these links are expanded upon, within the mathematical domain, by the human-specific ability to draw metaphors between distinct domains, thus creating a cultural expanded concept of the "number line."

According to the neuronal recycling model, the very possibility of retraining these circuits to perform more advanced functions, such as the mental number line, may be dependent on the distance between the function that these circuits originally evolved to serve, and their use in abstract reasoning, such as in the case of mathematics. Thus, certain mathematical concepts such as the concept of zero, of negative numbers, or of complex numbers may be difficult to grasp because they require an important reorganization of the internal representation of numbers, associated with a considerable amount of neuronal recycling. To take a more advanced mathematical example, fractal objects such as Cantor dust, the Koch snowflake, and the Sierpinski gasket may be difficult to understand because they violate the normal connection between area and perimeter that may constitute a strong evolutionary expectation built into the very structure of our parietal circuitry for object properties in space. By better understanding the neural foundations that make such abstractions possible, we may come to a deeper appreciation of the drastic reorganization necessary to attain such mathematical insights and may be able to develop better methods for explaining and teaching such profound mathematical ideas to children.

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