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Developmental trajectory of number acuity reveals a severe impairment in developmental dyscalculia

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ABSTRACT

Developmental dyscalculia is a learning disability that affects the acquisition of knowledge about numbers and arithmetic. It is widely assumed that numeracy is rooted on the "number sense", a core ability to grasp numerical quantities that humans share with other animals and deploy spontaneously at birth. To probe the links between number sense and dyscalculia, we used a psychophysical test to measure the Weber fraction for the numerosity of sets of dots, hereafter called number acuity. We show that number acuity improves with age in typically developing children. In dyscalculics, numerical acuity is severely impaired, with 10-year-old dyscalculics scoring at the level of 5-year-old normally developing children. Moreover, the severity of the number acuity impairment predicts the defective performance on tasks involving the manipulation of symbolic numbers. These results establish for the first time a clear association between dyscalculia and impaired "number sense", and they may open up new horizons for the early diagnosis and rehabilitation of mathematical learning deficits.

1. Introduction

Humans and many other animal species have evolved a capacity to represent approximate number. This "number sense" is at the heart of the preverbal ability to perceive and discriminate large numerosities (Feigenson, Dehaene, & Spelke, 2004) and relates to the intraparietal sulcus, a brain area which contains neurons tuned to approximate number in the macaque monkey and which is functionally active already at 3 months of age in humans (Izard, Dehaene-Lambertz, & Dehaene, 2008; Nieder & Miller, 2004; Piazza, Izard, Pinel, Le Bihan, & Dehaene, 2004).

Children discriminate numerosities long before language acquisition and formal education, as early as at 3 hours after birth (Izard, Sann, Spelke, & Streri, 2009). However, numerosity discrimination improves from a ratio of 1:2 to 2:3 before the year of age (Lipton & Spelke, 2003; Xu, Spelke, & Goddard, 2005) and undergoes progressive refinement throughout childhood (Halberda & Feigenson, 2008). The approximate number system is thought to encode numerosities as analog magnitudes (Dehaene, Piazza, Pinel, & Cohen, 2003, for review), that can be modeled as overlapping Gaussian distributions of activations on a logarithmically compressed internal continuum (Izard et al., 2008; Piazza et al., 2004). Logarithmic compression implies that the overlap between numbers increases with

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magnitude, which in turns decreases their discriminability (in obeyance to Weber's Law). However, discriminability critically depends on the width of the Gaussian distributions. The width of the distribution, referred to as the "internal Weber fraction" (Izard et al., 2008; Piazza et al., 2004), measures the precision of the internal representation and is therefore a sensitive index of "number acuity". The Weber fraction changes radically from infancy to adulthood (Halberda & Feigenson, 2008; Pica, Lemer, Izard, & Dehaene, 2004; see Piazza & Izard, 2009, for review). Moreover, its variability across individuals in a group of 14-year-old children was found to correlate with math achievement scores obtained across the previous school years (Halberda, Mazzocco, & Feigenson, 2008), thereby providing support to the hypothesis that the approximate number system is foundational of numeracy skills.

Recent psychophysical data show that numerosity perception is susceptible to adaptation, suggesting that approximate number is a primary visual property like color, size, or frequency (Burr & Ross, 2008): we would directly perceive a set as containing approximately five objects ("fivish") much like we perceive an object as "red" or as "big". Pursuing this metaphor, color blindness, or the inability to perceive differences between some of the colors that the majority of people can distinguish, might have its analog in the numerical domain in terms of an inability to perceive differences between some numerosities that other people can distinguish, that is, a lower number acuity. This impairment, in turn, would lead to developing a specific weakness in the number domain, ultimately leading to developmental dyscalculia, a specific learning disability that affects the acquisition of knowledge about numbers and arithmetic (e.g., Geary, 1993; Landerl, Bevan, & Butterworth, 2004; Shalev, Manor, & Gross-Tsur, 2005). Dyscalculia is much less recognized and understood than dyslexia, despite its similar prevalence estimates (around 5%) and its invalidating consequences (Butterworth, 2005; Shalev, 2004, for reviews).

However, the link between number sense (i.e., the approximate number system) and dyscalculia has remained elusive. No studies to date have unambiguously shown that dyscalculic children are impaired in purely non-verbal and non-symbolic numerical tasks. In fact, contrary to this hypothesis, two studies have shown that dyscalculic children are impaired in comparing numerical quantities when represented by symbols (i.e., Arabic digits) but not when presented as non-symbolic numerosities (Iuculano, Tang, Hall, & Butterworth, 2008; Rousselle & Noel, 2007). It therefore seems possible that a subtype of dyscalculia involves a partial disconnection or lack of automatic conversion between number symbols and the corresponding quantities. It is also possible, however, that non-symbolic numerosity processing was not measured finely enough in dyscalculic children for a deficit to become apparent.

Here, we probe this issue using a quantitative, psychophysical method for estimating the precision of the internal numerosity representation (i.e., number acuity) using a simple test devoid of any verbal and symbolic content with which we compared dyscalculic children to age and IQ matched typically developing children. Moreover, in order to quantify the impairment of dyscalculic children in this task we tested typically developing individuals of different age groups (both younger and older compared to the group of dyscalculics) in order to reconstruct the typical developmental trajectory of number acuity. Finally, in order to investigate whether the individual differences in the approximate number system acuity might give rise to individual differences in symbolic number processing ability in dyscalculic children, we correlated number acuity to their scores on symbolic number tasks from the diagnostic battery.

2. Methods

2.1. Participants

We tested typically developing participants of three age ranges (kindergarteners, school-age children, and adults) and a group of dyscalculic children matched in age and IQ with the typically developing school-age children. Participants, or their parents or legal representatives, gave informed consent.

2.1.1. Dyscalculic children

Twenty-five dyscalculic children were selected from a sample of children referred to a Child Neuropsychiatric Unit in Bergamo (Italy) because of specific learning disability in the mathematical and/or reading domains. Referral to a neuropsychiatric unit is the standard procedure in Italy when teachers suspect that poor achievement is not a consequence of social or motivational factors. A learning disability is diagnosed only after receiving a complete medical, psychological, neuropsychological and cognitive assessment carried out by an interdisciplinary team of psychologists, neurologists and speech therapists. The clinical diagnosis of dyscalculia was made through a widely used age-standardized Italian battery for developmental dyscalculia (Biancardi & Nicoletti, 2004; see Appendix A for further details). Further selection criteria were: (a) general intelligence, as measured by the Wechsler Intelligence Scale for Children – Revised (Wechsler, 1986), within the normal range; (b) normal or corrected-to-normal vision and hearing; (c) normal schooling; (d) no neurological and/or psychiatric disorder; (e) no attention-deficit hyperactivity disorder (ADHD), as evaluated through DSM-IV diagnostic criteria (American Psychiatric Association, 1994); and finally (f) no spoken language impairment.

Because of the well known high co-morbidity with dyslexia, children were also administered an age-standardized Italian test for developmental dyslexia (Sartori, Jorb, & Tressoldi, 1995). Twelve out of twenty-five children had scores below the cutoff point (two standard deviations in at least two out of the four sub-measures) and were therefore diagnosed also as dyslexic. Importantly, however, dyslexic-dyscalculic children did not differ from non-dyslexic-dyscalculic children on any of the measures of interest (age, general intelligence, dyscalculia battery scores, and more importantly number acuity (w); all Ps > 0.2).

Dyscalculics were tested over two sessions (separated by an average 2-months interval). WISC-R, dyscalculia

Table 1
Age-standardized scores for the dyscalculic and the non-dyscalculic children on the sub-tests of the dyscalculia and the dyslexia diagnostic batteries for which we had scores for both groups.

	Sub-tests from the WISC-R				Sub-tests from the dyscalculia battery								Sub-test from the dyslexia battery	
	Similarities		Figure completion		Number reading[a]		Multiplication tables[a]		Simple additions[a]		Complex written calculation[a]		Word reading accuracy[a]	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Dyscalculics ($N = 23$)	11.96	2.67	12.22	2.31	−7.5	8.79	−3.22	1.53	−2.61	2.11	−2.19	2.11	−2.27	2.02
Controls	13.04	1.75	12.15	2.31	.43	.32	.59	.43	.27	.47	.50	.81	.54	.42

[a] Significant difference across groups (all $P < 0.001$).

and dyslexia batteries were administered in the initial screening session. In the second session we administered the numerosity comparison task. Two children were excluded from the analyses because the R^2 of the sigmoidal fit for the number acuity test was inferior to 0.2. The final group consisted in 23 children with a mean age of 10.69 (range 8.25–12.58) and average Full IQ of 106 (standard deviation (SD) = 10; Verbal IQ = 108, SD = 12; Performance IQ = 102, SD = 11).

2.1.2. Non-dyscalculic controls (school-age children)

Twenty-nine children from a public primary school in north-eastern Italy, recommended as normal calculators and readers by their teachers, were tested individually by an experimenter in a quiet classroom. The children were selected to match for chronological age and general intelligence the dyscalculic children. Together with the main test on numerosity comparison, other tests were administered to assess their general intelligence, calculation and reading abilities. Scores in the WISC-R's Similarities and Figure Completion sub-tests were used as measures of verbal and non-verbal IQ, respectively. Calculation abilities were measured using four sub-tests of the dyscalculia battery (number reading, multiplications, simple additions, and written calculation), while reading abilities were assessed by means of the dyslexia battery (see Table 1 for the means and SD of all common tests across groups). Three children were excluded from further analysis because the R^2 of the fitting procedure used to calculate number acuity was very low (inferior to 0.2). The final group consisted in 26 children (mean age 10.43, age range 8.58–12.17) and it matched to the group of dyscalculics for both age ($t_{47} = 0.77$, $P = 0.44$) and IQ ($t_{47} = -1.69$, $P = 0.1$, $t_{47} = 0.09$, $P = 0.9$ for the Similarities and the Figure Completion sub-tests respectively).

2.1.3. Kindergarteners

Forty-four children from two different public kindergarten schools in north-eastern Italy were tested individually by a trained teacher, during school hours, in a quiet classroom, for about a quarter of an hour. Eighteen children were excluded from further analysis because either the fitting procedure used to calculate number acuity did not converge (indicating quasi-random responses) or the R^2 of the fit was inferior to 0.2. The final group consisted of 26 children (mean age 5.25, age range 3.8–6.2).

2.1.4. Adults

Twenty adult participants (mean age 26.65, age range 22–33), with normal or corrected-to-normal vision, were tested in quiet testing rooms at the University of Padova.

2.2. Task and procedure

2.2.1. Non-symbolic number comparison

The stimuli were pairs of arrays of black dots presented in two white discs on either side of a central white fixation spot (Fig. 1). On each trial, one of the two arrays contained either 16 or 32 dots (reference numerosity, hereafter $n1$). The paired array for the 16 dot reference contained 12, 13, 14, 15, 17, 18, 19 or 20 dots (hereafter $n2$). For the 32 reference, numerosities for the second array were twice as large as those for Refs. 16 (24, 26, 28, 30, 34, 36, 38 and 40), for a total of 16 experimental conditions. Perceptual variables were randomly assigned to each stimulus pair such that, on average, on half the trials dot size of the non-reference numerosity array ($n2$) was held constant, and on the other half, the size of the area occupied by the non-reference numerosity array ($n2$) was held constant; in the reference numerosity arrays ($n1$), these parameters were varied simultaneously. This design was adopted to prevent participants from basing their performance on these non-numerical parameters (see Dehaene, Izard, and Piazza (2005), for a more detailed description of the logic behind such manipulations).

Participants were asked to decide without counting which array contained the largest number of dots by press-

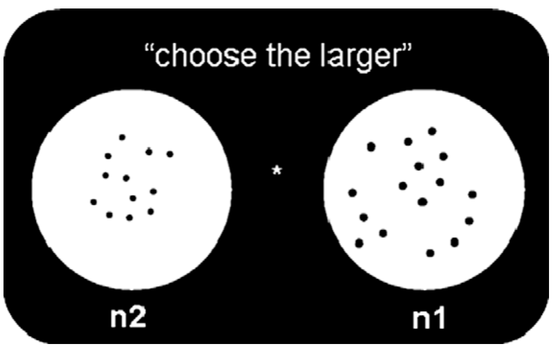

Fig. 1. Example of stimuli in the non-symbolic numerosity comparison task. Participants were shown pairs of arrays of dots on a computer screen and asked to decide which array contained more dots.

ing the corresponding button on the computer keyboard. The arrays remained on the screen until subjects gave their response. The experiment started with four training trials followed by a total of 80 experimental trials (five trials per condition). For 14 out of the 25 dyscalculic children, four $n2$ numerosities were added at the extremes of the distributions to obtain better estimates of the internal Weber fraction (see below): 10 and 22 for the arrays paired with $n1 = 16$, and double those values for the arrays paired with $n1 = 32$. The addition of such extreme values increases the probability of observing the full distribution of responses (from 0% to 100% response 'larger') and thus of better fitting the data with sigmoid functions. Moreover, in this modified design we increased the number of trials per condition, from 5 to 7 (for a total of 140 experimental trials). However, these modifications did not result in a significant change neither in R^2 of the sigmoidal fit, nor in the estimated w across subjects ($t_{33} = -1.06$, $P = 0.29$, and $t_{33} = -0.73$, $P = 0.47$, respectively). The results from the two experiments where thus pulled together. This long version of the task was also administered to the age-matched controls.

3. Results

3.1. Developmental trajectory of number acuity in typically developing subjects

In all three age groups of typically developing subjects (kindergarteners, school-age children and adults), as expected, "larger" responses to $n2$ followed a classic sigmoid curve. The slope was approximately twice as large for trials where the stimuli were twice larger, thus replicating earlier findings of Weber's law for numbers (Piazza et al., 2004) (Fig. 2A). The curves became parallel when plotted on a log scale (Fig. 2B), and identical once expressed as a function of the log ratio of the two numbers, again in excellent agreement with previous theorizing (Fig. 2C). Across age ranges, the slope of the central portion of the sigmoid

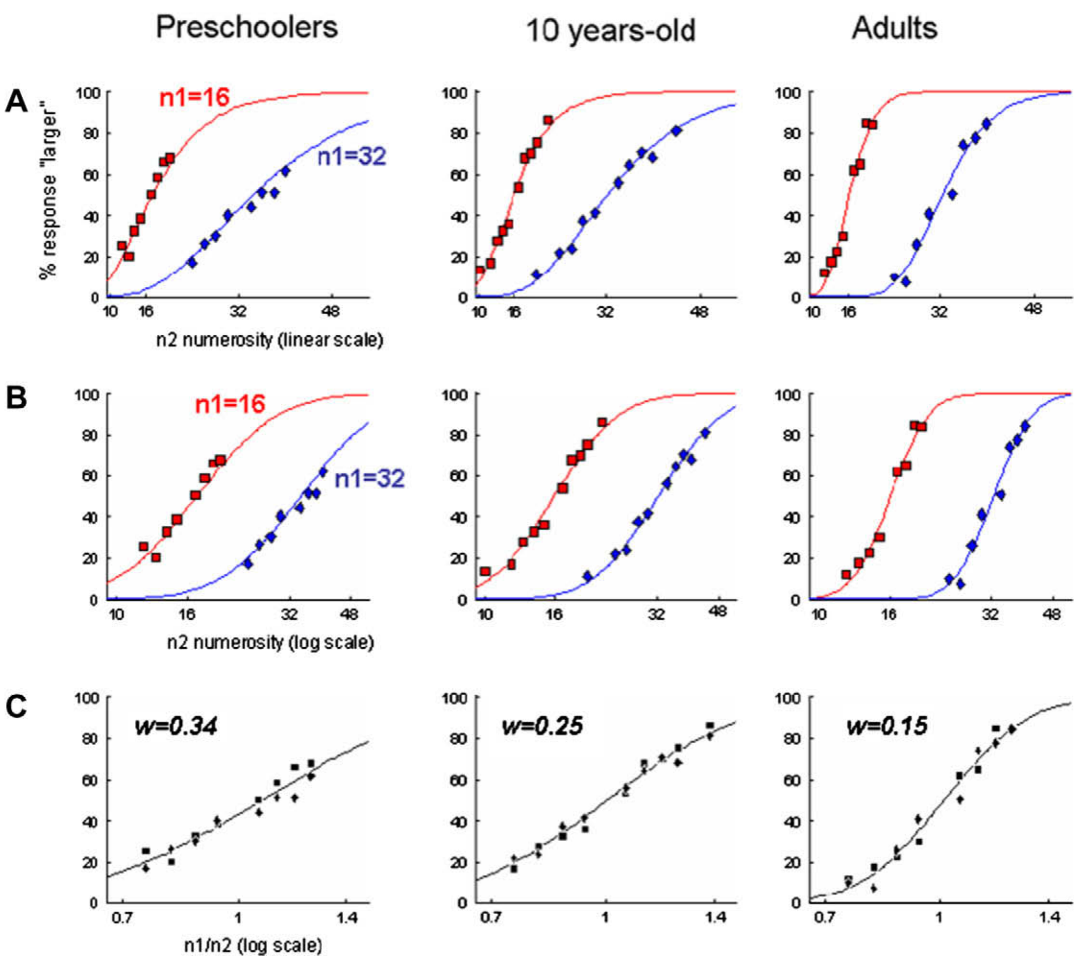

Fig. 2. Performance in the non-symbolic numerosity comparison task as a function of age group in typically developing participants. Graphs represent the proportion of trials in which participants responded that $n2$ was more numerous than $n1$. Performance is plotted as a function of $n1$. Performance curves assume a skewed shape when $n2$ is plotted on a linear scale (A) but become symmetrical on a logarithmic scale (B) and depend on the logarithm of the ratio of the two numbers (C). The fitted curves are derived from the equations that are described elsewhere (Piazza et al., 2004).

became steeper, indicating a progressive refinement in the internal representation of numerosity during the life-span (compare the three columns in Fig. 2).

In order to quantify this developmental pattern, for each participant, response distributions were used to estimate a measure of the precision of underlying numerical representation, the internal Weber fraction (thereafter *w*). This measure corresponds to the standard deviation of the estimated Gaussian distribution (on a log scale) of the internal representation of numerosity that generates the observed performance (a method previously described in the Supplemental Data from Piazza et al., 2004). Thus, 2*w* represents the percentage difference between two numerosities that is necessary to perceive them as different with ~95% confidence. To give a more concrete example, a *w* of 0.3 implies that in order for two sets to be easily discriminable they need to differ by about 60%, as in 10 vs. 16.

As expected, mean *w* decreased with age group ($F_{2,69} = 19.95$, $P < 0.001$), starting from an average of 0.34 for the kindergarteners, down to 0.25 for the 10 year old children ($t_{50} = 2.99$, $P = 0.005$), and 0.15 for the adults ($t_{44} = 4.46$, $P < 0.001$). These results are consistent with recent observations from Halberda and Feigenson (2008) reporting, in a comparison task similar to the one used in the present study, an estimated internal Weber fraction of 0.38 for the kindergarteners and of 0.11 for adult participants (compared to 0.34 and 0.15 in the present study).

This developmental trend can be well described by a power function with a negative exponent of −0.5 (least squares fitting $r^2 = 0.97$), indicating an initial sharp decay followed by a progressively smaller but long-lasting reduction in time (see Fig. 3). Because the rate of refinement of the estimated internal Weber fraction is high during the initial years of life, we wondered if we could observe an effect of age even within our relatively small sample of kindergarteners. Thus, we separated the kindergarteners into young (mean age 4, 2), medium (mean age 5, 2), old (mean age 5, 9) and compared the average *w* across groups. As expected, we observed a reduction of *w* across age ranges (see Fig. 3), although, due to the small sample size of each group (6, 9, and 11 children in each group), only the difference between the young and the old reached statistical significance ($t_{15} = 2.22$, $P = 0.04$).

3.2. Number acuity in developmental dyscalculia

We investigated number acuity in dyscalculic children by relating their performance to that of typically developing children. First, we tested the hypothesis that dyscalculic children had impaired number acuity compared to age and IQ matched controls. Second, we quantified the size of their impairment by relating their performance to the developmental trajectory of typically developing children. Finally, we relate the size of number acuity impairment to performance in symbolic number tasks.

Similarly to controls, response distribution in the numerosity comparison task followed Weber's law: the slope was approximately twice as large for trials where the stimuli were twice larger, and became identical once expressed as a function of the log ratio of the two numbers. However, group comparison of the individual subjects' estimates of number acuity indicated a highly significant impairment in dyscalculics ($w = 0.34$) compared to controls ($w = 0.25$, $t_{47} = 2.90$, $P = 0.003$). Thus, number acuity in dyscalculic children deviated from the normal developmental trajectory (see Fig. 3). Moreover, although number acuity is calculated from response accuracy, reaction times (hereafter RTs) were also collected during the psychophysical task. Mean RTs significantly decreased with age in the non-dyscalculic groups ($F_{2,69} = 61.24$, $P < 0.001$), starting from an average of 2524 ms for the kindergarteners, down to 1314 ms for the 10–year-olds ($t_{50} = 7.81$, $P < 0.001$), and 1016 ms for the adults ($t_{44} = 2.93$, $P = 0.005$). Thus, RTs followed a developmental trend that was highly similar to the one observed for *w*, well described by a power function with a negative exponent of −0.53 (least squares fitting $r^2 = 0.91$) (see Fig. 4). Interestingly, despite the fact that dyscalculic and controls differed in numerical acuity, they did not differ in terms of RTs (1271 ms vs. 1314 ms, respectively; $t_{47} = 0.39$, $P = 0.69$). These results indicate that the poorer acuity of dyscalculic children in comparison to control children does not stem from the use of different response strategies but reflects differences in the internal representations.

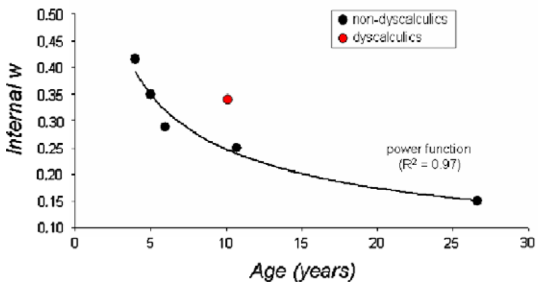

Fig. 3. Developmental trajectory of *w*. The graph represents mean *w* as a function of mean age and group. In black, developmental trajectory for the non-dyscalculic group. In red, mean *w* for the dyscalculic group, whose performance is equivalent to that of 5 years younger non-dyscalculic controls. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

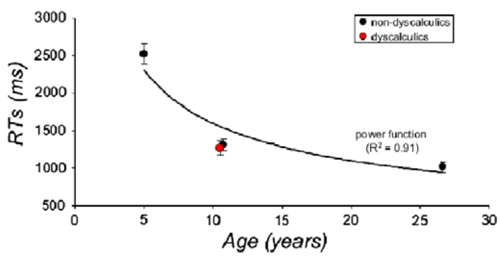

Fig. 4. Developmental trajectory of RTs in the number acuity test. The graph represents mean RTs as a function of mean age and group. In black, developmental trajectory for the non-dyscalculic group. In red, mean RTs for the dyscalculic group, whose performance is equivalent to that of age matched non-dyscalculic controls. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

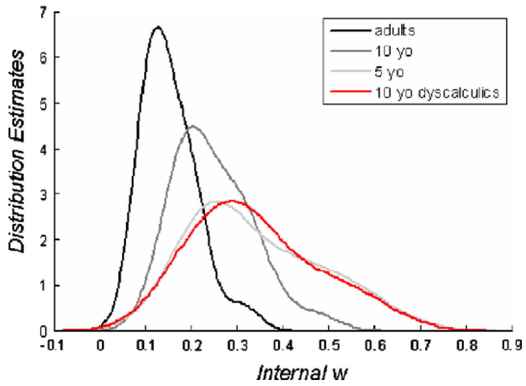

Fig. 5. Distribution estimates of *w* as a function of age group. In black and gray distribution estimates of *w* in the non-dyscalculic groups. In red, distribution estimates of *w* in the dyscalculic group. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

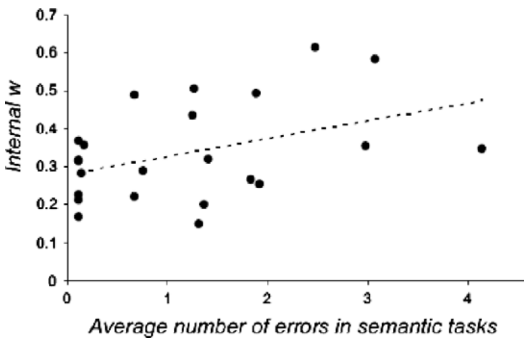

Fig. 6. Non-symbolic number acuity (*w*) predicts accuracy in manipulating numerical symbols amongst dyscalculic children.

In order to quantify the size of the impairment in number acuity, we compared the mean *w* in the dyscalculic group to the mean *w* in the typically developing kindergarteners, who were on average 5 years younger. Results showed that the two groups did not differ in number acuity ($t_{47} = 0.06$, $P = 0.95$) (see Fig. 5 for the distribution of *w* in all tested groups), suggesting a 5 years delay in basic non-symbolic quantity processing for dyscalculic children.

In order to more directly explore the relationship between an impaired core sense of number and the impairments in symbolic numerical skills in dyscalculic children, we calculated the correlations between *w* and the averaged accuracy scores of different groups of tests of the dyscalculia battery (see Appendix A for details). We grouped the tests into four groups according to the types of skill that they investigate (cf. Dehaene et al., 2003): transcoding skills (reading, writing, and repeating numbers), quantity and relational-based skills (so called "semantic abilities" consisting in different forms of number comparison), simple arithmetical facts retrieval skills (multiplication tables, very simple additions and subtractions), and complex written and oral calculation skills (additions and subtractions including two and three digit numbers' operands). Of particular interest for the present research are the semantic tests because they directly hinge upon the representation and manipulation of numerical quantities without invoking knowledge of arithmetic. In those tests children were asked to compare Arabic digits either by choosing the largest among three numbers (e.g., "what is the largest number?": 12 54 23), or by positioning numbers in one of four possible positions among three other numbers (e.g., "where is number 10?": *X* 5 *X* 8 *X* 15 *X*). In order to succeed in these tests, a child must know what a given numerical size means, and must be able to appreciate and compute the proximity relations between different numbers. This knowledge is definitional to the semantic of numbers.

Our hypothesis was thus that (non-symbolic) number acuity would predict scores in (symbolic) semantic tests and also in complex calculation because, according to a widely accepted neurocognitive model of number processing (Dehaene et al., 2003) these tasks (but not transcoding nor simple arithmetic fact retrieval) draw upon the core quantity system. Indeed, semantic scores were reliably predicted by our index of (non-symbolic) number acuity ($R^2 = 0.17$, $F_{1,22} = 4.371$, $P = 0.049$) (see Fig. 6). Importantly, number acuity still explained a significant proportion of variance of the semantic scores even after partialling out the effects of age and verbal IQ, which may be important but non-specific variables (hierarchical regressions testing the significant additional portion of variance explained by number acuity after having regressed out the impact of age

Table 2
Full correlation matrix of *w*, the average accuracy in the four sub-tests of the developmental dyscalculia battery, and word reading accuracy.

		w	Semantic tests	Transcoding tests	Simple calculation tests	Complex calculation tests	Reading ability
W	Pearson's *r*	1	.414	.163	−.090	.076	.019
	P-value		.049	.458	.684	.729	.929
Semantic tests	Pearson's *r*		1	.460	.373	.512	.246
	P-value			.027	.080	.013	.257
Transcoding tests	Pearson's *r*			1	.110	.120	.048
	P-value				.618	.586	.827
Simple calculation tests	Pearson's *r*				1	.772	.458
	P-value					.000	.027
Complex calculation tests	Pearson's *r*					1	.320
	P-value						.136
Reading ability	Pearson's *r*						1
	P-value						

and IQ: R^2 change = 0.24, F change$_{1,19}$ = 6.11, P = 0.02). Importantly, number acuity was not simply a generic predictor of performance because, in other tasks not involving number processing, such as word reading, number acuity was not a good predictor (R^2 = .00, $F_{1,22}$ = 0.01, P = 0.93). Even in the number domain, as expected, *w* did not predict transcoding performance (R^2 = .03, $F_{1,22}$ = 0.57, P = 0.46) nor simple arithmetical facts retrieval performance (R^2 = .01, $F_{1,22}$ = 0.17, P = 0.68). Surprisingly, however, and in contrast with our initial hypothesis, *w* did not predict calculation performance either (see Table 2 for the entire correlation matrix).

4. Discussion

The typical developmental trajectory of non-symbolic number acuity showed an initial sharp decay followed by a progressively smaller but long-lasting reduction in time, in agreement with a recent report (Halberda & Feigenson, 2008). The cause of the reduction of *w* with age remains still unknown. It could reflect purely maturational processes, but also a contribution from the teaching of counting and arithmetic which typically starts at the end of kindergarden. However, two observations seem to favour the maturational hypothesis, at least across the first few years of life. First, the developmental trend that we report in the present study, well described with a power function, is also coherent with previous observations on babies, where the critical ratio needed for numerosity discrimination was found to decrease dramatically within the first year of life from 3:1 soon after birth (Izard et al., 2009), to 2:1 at 6 months and down to 3:2 at 9-months (Lipton & Spelke, 2003). It is unlikely that the developmental trend observed within the first year of life can be accounted for by cultural or educational factors. Second, the estimate of *w* in adults (0.15) is consistent with, although slightly lower than, the value observed in adults of the remote Munduruku culture where the number lexicon is restricted to numerals up to 5 and where children do not undergo formal teaching of counting and arithmetic (Pica et al., 2004). Our initial observations, in a population with radically different cultural background, argue against a major effect of education and culture on the observed refinement in number acuity, although we cannot exclude that its final adult value is partially influenced by experience with symbolic numerals.

Importantly, the present research shows that number acuity is severely impaired in dyscalculic children in comparison to their normally developing peers (matched for age and general intelligence). This finding fits with neuroimaging studies showing that regions of the parietal cortex, close to the locus of specialised neural systems for numerical quantity (Cantlon, Brannon, Carter, & Pelphrey, 2006; Piazza, Pinel, Le Bihan, & Dehaene, 2007; Piazza et al., 2004) are structurally or functionally impaired in dyscalculic subjects (Isaacs, Edmonds, Lucas, & Gadian, 2001; Molko et al., 2003, 2004; Price, Holloway, Rasanen, Vesterinen, & Ansari, 2007), and supports the notion of a foundational role of the approximate number system in the development of symbolic numerical abilities (Halberda et al., 2008). Our study, moreover, because it traces the entire developmental trajectory of the approximate number system during the life-span, is able to quantify the size of number acuity impairment in dyscalculia in terms of a 5-years delay. It is for future study to understand if the development of the approximate number system in dyscalculic children deviates qualitatively or only quantitatively from the developmental trajectory of the normally developing children.

Finally, we have also shown that poor number acuity in dyscalculic children was directly reflected in their ability to perform simple numerical tasks over symbolic stimuli (Arabic numbers comparison tasks). Interestingly, and contrary to our initial hypothesis, number acuity did not predict calculation performance. A posteriori, we speculate that while individual differences in the number comparison tasks can reveal pure impairments in the semantic representation of numerical quantities, performance in calculation might be also modulated by the ability of children to elaborate and apply non-strictly number-related strategies. It is possible that, because dyscalculic children have an immature quantity representation system, when confronted with arithmetical problems they deploy compensatory strategies using ancillary systems. For instance, previous research has reported evidence for massive use of finger counting in dyscalculic compared with non-dyscalculic children (Jordan, Hanich, & Kaplan, 2003; Ostad, 1997, 1999). While, on average, such compensatory mechanisms do not allow dyscalculic children to perform as controls on arithmetical tasks, they may introduce additional across subjects variability. As a consequence, while normal children's performance in calculation tasks directly reflects the acuity of the core quantity representation system (Halberda et al., 2008), in dyscalculic children it might become increasingly dependent on the integrity of the ancillary systems that children recruit to solve the tasks. For instance, it has been suggested that poor working memory resources lead to difficulty in executing calculation procedures and may also affect learning of arithmetic facts (Geary, 1993). This, together with the relatively small sample size (N = 24) used in the present study, may explain why we failed to observe the expected correlation between *w* and performance in calculation tasks.

As a whole, these findings lend support to remediation programs for developmental dyscalculia that include exercises aimed at retraining the core non-symbolic sense of number and to cement its links to the symbols used to denote it (Wilson, Revkin, Cohen, Cohen, & Dehaene, 2006). It should be stressed that, like all correlational studies, the present work does not firmly establish the causality of the link between dyscalculia and impaired number acuity. The refined precision of the Weber fraction with age could reflect maturational processes, but also, at least partially, a contribution from the teaching of counting and arithmetic which typically starts in late kindergarten. Mathematical and neuronal network models suggest that the acquisition of number symbols may induce a radical change in the internal representation of numbers, with an increased precision (Dehaene, 2007; Verguts & Fias, 2005) and a shift from a logarithmic sense of approximate numerosity to a linear sense of exact number (Berteletti, Lucangeli, Piazza, Dehaene, & Zorzi, 2010; Dehaene, Izard, Spelke, & Pica, 2008; Siegler & Opfer, 2003). This refinement in numerical precision with exposure to symbols may not occur normally in dyscalculic children, thus leaving 10-years-old children with the same precision as a 5-year-old child. In that respect, our results remain compatible with two alternative interpretations of the causes of dyscalculia: either a pure impairment in non-symbolic number representation, or an inability to link this representation with the corresponding number symbols, which in turns influences the precision with which numerosity is perceived. A similar form of "spiral causality" exists in the domain of dyslexia, where lack of phonological awareness is seen both as a putative cause of the reading impairment (e.g., Ramus, 2003), but also as a consequence of learning to read in an alphabetical script (e.g., Castles & Coltheart, 2004).

Another important caveat is that deficits in other non-numerical domains may be associated with dyscalculia. Indeed, previous studies have pointed to impaired finger knowledge (Benson & Geschwind, 1970), working-memory (McLean & Hitch, 1999) and visuo-spatial abilities (Rourke & Conway, 1997). In the present study we did not test the aforementioned abilities, therefore we cannot know if our dyscalculic subjects were also impaired in those domains. Nevertheless, our study is the first to show and quantify a clear impairment in a core numerical system that is foundational for the development of higher level numerical skills (Gilmore, McCarthy, & Spelke, 2007; Halberda et al., 2008). Future studies should directly compare the strength of the associations between mathematical achievement and the abilities related to all cognitive domains that have been shown to be impaired in dyscalculics.

Keeping these limitations in mind, our results demonstrate how a very simple psychophysical measurement of non-symbolic numerical acuity can provide a quick and efficient tool for evaluating the nature and depth of the deficit in dyscalculic children.

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Appendix A

A.1. Description of the dyscalculia battery

Developmental dyscalculia was assessed in the present study through a widely used Italian age-standardized battery (Biancardi & Nicoletti, 2004). This battery is based on several tests from which accuracy and/or total time are recorded. For diagnostic purposes, the tests' scores are grouped in two main sub-quotients, the "numerical quotient" and the "calculation quotient". The "numerical quotient" is based on the averaged scores from the following tests:

1. *Arabic number reading*: The child reads aloud 12 Arabic numbers (3–6 digits). Both accuracy and speed are measured.
2. *Arabic number writing*: The child writes in Arabic format 12 spoken words (3–6 digits) named by the experimenter. Accuracy is measured.
3. *Repetition:* The child repeats 12 spoken number words (3–6 digits) named by experimenter. Accuracy is measured.
4. *Triplets*: For 20 trials the child chooses the largest number among a set of three Arabic numbers (1–6 digits). Both accuracy and speed are measured.
5. *Insertions*: For 12 trials the child positions a number (1–5 digits) in one of four possible positions among three other numbers (e.g., "where is number 10?": *X* 5 *X* 8 *X* 15 *X*). Both accuracy and speed are measured.

The "calculation quotient" is based on the averaged scores from the following tests:

1. *Simple calculation*: The child performs 16 multiplications (operands between 1 and 9, e.g. 6×3; 8×1; 2×5), 6 additions and 6 subtractions with results smaller than 10, e.g. $4 + 2$; $5 + 3$; $5 - 2$; $7 - 5$. A response is scored as correct only if it was also given within a 2 s deadline. Accuracy is measured.
2. *Complex oral calculation*: The child performs 10 additions and 10 subtractions with results above 10, e.g., $8 + 5$; $3 + 8$; $18 - 6$. A response is scored as correct only if it was also given within a 15 s deadline.
3. *Complex written calculation*: The child performs 12 written calculation problems (4 additions, 4 subtractions, 4 multiplications; e.g., $839 + 243$; $435 - 279$; 18×3; 23×41). The total time allowed for completion is 10 min. Accuracy is measured.

The scores from these two quotients are then age-standardized, and children are considered dyscalculic if their score is at least 2 SD below the average on at least one of the two quotients.

To perform correlations with *w*, however, we regrouped the sub-tests into four classes of tests, in order to be more sensitive to the different types of numerical skills that the different sub-tests tap onto (in the dyscalculia battery, for example, the "numerical quotient" merges together transcoding abilities and semantic number manipulation abilities, while the "calculation quotient" merges together simple facts retrieval abilities and complex calculation abilities). On the basis of our knowledge of the neurocognitive architecture of number processing (Dehaene, Piazza, Pinel, & Cohen, 2003) we thus averaged, for each child, the accuracy scores on the different tests into four groups:

1. *Transcoding and repetition tests*: This group includes Arabic number reading, Arabic number writing and Repetition.
2. *Semantic number coding tests*: This group includes Triplets and Insertions.
3. *Simple calculation tests*: This group comprises the Simple calculation tests.

4. *Complex calculation tests*: This group includes Complex oral and Complex written calculation tests.

A.2. Description of the dyslexia battery

Developmental dyslexia was assessed through a widely used Italian age-standardized battery (Sartori, Jorb, & Tressoldi, 1995). The ability to read aloud single words and nonwords (accuracy and fluency) was measured on a standardized list of 102 Italian words and 48 Italian nonwords. Reading fluency is measured by total time (in seconds) spent on a specific list. Children were classified as dyslexic if their performance was two SDs below the norm on at least two of the four available measures (i.e., accuracy and fluency for each of the two lists).

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