# OCCAM'S RAZOR IS NOT A SWISS-ARMY KNIFE: A reply to Pillon and Pesenti 

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Our paper reported a dissociation, in a pure alexic patient, between impaired reading and multiplication of Arabic digits on the one hand, and relatively preserved comparison, subtraction, addition, and simple division on the other hand (Cohen \& Dehaene, 2000). We showed how the main features of this puzzling case can be explained by the triple-code model of number processing (Dehaene \& Cohen, 1995; Dehaene, Dehaene-Lambertz, \& Cohen, 1998). (As a matter of historical fact, the model was actually used to predict this dissociation and to actively look for it in our sample of patients.) This case also confirmed a previous description of two other pure alexic patients with preserved number comparison in the face of grossly impaired reading (Cohen \& Dehaene, 1995), and closely related observations in a split-brain patient (Cohen \& Dehaene, 1996). Similar findings have been reported by others (Gazzaniga \& Smylie, 1984; McNeil \& Warrington, 1994; Miozzo \& Caramazza, 1998; Seymour, Reuter-Lorenz, \& Gazzaniga, 1994).

Although Pillon and Pesenti (2001, this issue; henceforth P\&P) dismiss our evidence as not even constraining theories of numerical cognition, they struggle to find an alternative theory compatible with the facts. Starting with the postulate that there must be alternative accounts of our data, they bring forward a paraphernalia of theoretical constructs that are specifically tailored, one by one, to fit our empirical data (in their words, they are "[not]
claiming that it could account for a larger range of observations than just this study"; this issue, p. 283). In the end, they reach what they claim is a viable alternative with "three functional components." Numbers being the issue, it is interesting to observe that their Figure 1 actually reveals a total of 6 main boxes, some comprising up to 6 sub-components, linked by no less than 13 arrows.

To keep this reply short, we shall not dwell here on the avowed lack of "fine specification" of P\&P's model. Rather, we briefly show why the explanations that they sketch for the dissociations observed in patient VOL do not work.

## READING DEFICIT AND PRESERVATION OF OTHER NUMERICAL TASKS

1. $\mathrm{P} \& \mathrm{P}$ claim that the reading deficit resulted from an impaired access to Core Numeral Meaning, all subsequent processing being spared. However, their model includes an alternative route for semantic access, from the Arabic Numeral Recognition System to the Stored Arithmetic Knowledge, and thence to the Core Numeral Meaning. This intact pathway should allow for a successful reading, while the patient actually showed a severe reading deficit.

Indeed, in P\&P's framework, this path from the Stored Arithmetic Knowledge to the Core Numeral Meaning is demonstrably operative since it is the

[^0]pathway that is assumed to be used whenever the patient utters the correct result of subtraction problems whose operands were misread. Hence, at the very least, $\mathrm{P} \& \mathrm{P}$ should not expect operand reading errors to be more frequent than subtraction errors. However, the data show that operand reading errors were actually about twice as frequent as subtraction errors.

What this analysis shows is that P\&P's model is underspecified: they must explain why the patient sticks to using her impaired direct naming route while a preserved indirect route is available. Because our triple-code model also comprises multiple routes, it faces a similar problem. However, we have directly confronted this difficulty by specifying which routes are involved in various tasks, and in particular why we think that the direct asemantic visual-to-verbal path is the default route for reading (Cohen \& Dehaene, 1995); and contrary to P\&P's assertion, we have proposed a detailed model of this process in a previous publication (Cohen \& Dehaene, 1991).
2. $\mathrm{P} \& \mathrm{P}$ claim that reading aloud (without counting) was more severely impaired than number comparison only because, although both tasks rely on quantity representation, comparison tolerates some loss of precision at this level, whereas reading does not. This hypothesis entails that comparison errors should set an upper bound to reading errors. In other terms, the patient should not make more numerous or more severe reading errors than expected on the basis of her comparison performance. Actually, when reading pairs of numerals, VOL made significantly more errors reversing the relative size of the two numbers than would be predicted from her flawless comparison performance. This was also found in our previous report of two other pure alexic patients (Cohen \& Dehaene, 1995).

Similarly, although we did not ask VOL to compare numbers differing by one unit, we know for certain that she was flawless when comparing twodigit numbers differing by two units. This implies that the quantity representation that she derived from Arabic numerals was inaccurate by less than two units. This would predict, according to P\&P's model, that her reading errors should also be off the
target by no more than a few units. This was not the case, as she commonly made gross reading errors, such as $83 \rightarrow 94$ or $65 \rightarrow 92$.
3. $\mathrm{P} \& \mathrm{P}$ claim that the patient's recognition system for arabic numerals, but also for verbal numerals, was intact (P\&P's Figure 1). Although we did not specifically study the processing of spelled-out numerals, the idea that processing verbal numerals was unimpaired in a patient with pure alexia who was almost unable to read any printed word is extremely implausible. Reading spelled-out numerals was severely impaired in our previous two cases of pure alexia (Cohen \& Dehaene, 1995).
4. P\&P claim that the arabic-to-semantic conversion process was impaired in such a way that rank information was normally accessed, but that quantity information was accessed in only an approximate fashion. This hypothesis would account for the sparing of reading-by-counting while attributing the reading impairment to a failure of semantic access. However, the model depicted in P\&P's Figure 1 includes a single common access route to both aspects of semantic knowledge (rank and quantity), which does not allow for such a dissociation. Furthermore, if the patient could indeed access accurate rank information, it is unclear why such information would not suffice to directly name the number, without having to recite the series of number names. If VOL knows that the shape " 6 " refers to the sixth number, why can't she immediately access the corresponding name "six"?

## ARITHMETIC DEFICIT AND DISSOCIATIONS BETWEEN OPERATIONS

1. $\mathrm{P} \& \mathrm{P}$ claim that an impaired access to Core Numeral Meaning from the Arabic Recognition System causes a deficit of arithmetic abilities. However, their model features (a) intact access to Stored Arithmetic Knowledge, and (b) intact bidirectional links between Stored Arithmetic Knowledge, Core Numeral Meaning, and Calculation Procedures. Hence, all components involved in arithmetic
problem solving should operate normally, and no arithmetic deficit should occur whatsoever.
2. $\mathrm{P} \& \mathrm{P}$ claim that, according to their account of the patient's deficit, simple familiar problems should be spared, while only unfamiliar problems requiring semantic elaboration, should be impaired. In Table 2, we report the patient's performance with a set of elementary multiplication problems whose first operand was 1 or 2 , and whose second operand was less than 9 (i.e., multiplications such as $2 \times 3$ or $1 \times 5$ ). It seems indisputable that such problems are highly familiar. According to P\&P's theory, therefore, they should have been solved flawlessly. As shown in Table 2, the patient actually made $88.9 \%$ errors.
3. $\mathrm{P} \& \mathrm{P}$ claim that their theory accounts for the dissociation between operations. However, if indeed unfamiliar arithmetic facts were more vulnerable in patient VOL due to a weaker representation in Stored Arithmetic Knowledge, one would expect a better performance for the overlearned multiplication facts than for the corresponding subtraction facts. This prediction is exactly opposite to the highly significant dissociation that was observed.
4. $\mathrm{P} \& \mathrm{P}$ claim, if we understand their proposal correctly, that the patient tried to compensate for her difficulties in multiplication by resorting to a "second pass" through the number processing system: She first named (erroneously) the operands, then fed this verbal output into her Verbal Numeral Recognition System, and eventually accessed the whole of her numerical knowledge. This device allows $\mathrm{P} \& \mathrm{P}$ to account for the fact that the results that were proposed by VOL to multiplication problems were correct relative to her reading of the operands. First, this contrived procedure would amount to nothing more than recycling erroneous operands into the number system, with no reason to expect any improvement in the resolution of the target problem. Second, P\&P propose no principled reason why this strategy should apply to multiplication but not to other operations.

## CONCLUSION

We conclude that P\&P's model does not seem to meet its authors' expectations as an alternative account of patient VOL's behaviour. Although we look forward to the next generation of number processing theories, the triple-code model still seems to provide a cogent, if simplified, account of most, if not all, reported cases.

From an epistemological point of view, we concur with $\mathrm{P} \& \mathrm{P}$ in thinking that a theory is never validated by the accumulation of compatible empirical facts. It merely remains sustainable until a sufficient amount of contradictory evidence has piled up to reject it, and until a viable alternative has seen the light. P\&P propose that, before a single-case study can be accepted as evidence in favour of a theory, all alternative accounts should be rejected. This is of course desirable, but the space of possible theories is infinite. P\&P’s precept states: "to give support to your theory (...) show that the opposite view cannot work" (this issue, p. 283). What is the "opposite" of a theory, however? Should all possible views, however far-fetched and unmotivated by prior evidence, be considered?

We think that two principles must guide the development of a solid theoretical framework. First, converging data sets from multiple studies and methods must be used to constrain the theory. The triple-code model was proposed based on a thorough review of the neuropsychology of number processing, including single-case studies, group studies, brain-imaging data and cognitive psychological experiments. Second, Occam's principle must be used to avoid unnecessary multiplication of theoretical constructs. Some of P\&P's proposals, such as the separation of the semantic component into rank and quantity information, seem potentially fruitful and may ultimately be incorporated in a revision of the triple-code model. However, the current evidence that supports this hypothesis is still lacking.

## COHEN AND DEHAENE

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