BRAIN AND LANGUAGE 47, 214–232 (1994)

Amnesia for Arithmetic Facts: A Single Case Study

LAURENT COHEN

Service de Neurologie, Hôpital de la Salpêtrière, Paris, France

AND

STANISLAS DEHAENE

Laboratoire de Sciences Cognitives et Psycholinguistique, INSERM & CNRS, Paris, France

We report the case of an anarithmetic patient with a selective deficit of memory for elementary arithmetic facts, who produced, for instance, "25" in answer to "4 × 5." The patient showed good language comprehension and production abilities, had minimal number transcoding difficulties, and mastered normally multidigit arithmetic procedures. She was submitted to a series of calculation, verification, and number classification tasks. The arithmetic deficit was evident in both recognition and recall tasks, was consistent across testing sessions, and did not vary as a function of the format used for presentation of the problems. The patient failed even when only implicit access to arithmetic facts was expected: In a timed addition verification task, she did not show a normal inhibition effect when rejecting addition problems in which the proposed result was the product of the two operands (e.g., "3 + 4 = 12"). We suggest that the deficit resulted from a specific and permanent degradation of some connections and nodes in arithmetic long-term memory.

INTRODUCTION

Ever since Henschen (1919) isolated *acalculia* as an independent neuropsychological syndrome, a number of selective dissociations have been reported within the calculation system. Hécaen, Angelergues, and Houillier (1961) introduced the term *anarithmetia* to refer to patients whose calculation deficits could not be attributed to general linguistic or spatial

The authors gratefully thank Dr S. Bakchine for referring the patient, as well as Mrs D. Sosson and S. Jonas for their assistance. Address correspondence and reprint requests to Dr Laurent Cohen, Service de Neurologie, Clinique Paul Castaigne, Hôpital de la Salpêtrière, 47 Bd de l'Hôpital, 75651 Paris CEDEX 13, France.

0093-934X/94 $6.00

disorders. Caramazza and McCloskey (1987) further documented a dissociation between the knowledge of elementary addition and multiplication tables, and the procedures for performing complex multi-digit operations. The present article is dedicated to a case study of selective deficit of memory for simple arithmetic facts and how it may inform us about the structure of arithmetic memory.

McCloskey and his colleagues (e.g. McCloskey, Harley, and Sokol, 1991; Sokol, McCloskey, Cohen, and Aliminosa, 1991; McCloskey, Aliminosa, and Sokol, 1991; McCloskey, 1992) have published extensive and detailed studies of more than 15 cases of impaired memory for arithmetic facts. All of these patients were characterized by an inability to produce, aloud or in writing, the correct result of simple two-digit addition, subtraction, or multiplication problems (e.g., $3 \times 6 = ?$). In multiplication, which is most frequently impaired, error patterns shared several characteristics across patients (McCloskey et al., 1991):

(i) Multiplication by 0 and by 1 was often uniformly preserved or impaired, with systematic errors of the type $0 \times N = N$, suggesting a common rule-based representation for these facts.

(ii) By contrast other multiplication problems showed a non-uniform pattern of impairment (e.g., impairment for 8×8 with preservation of 8×9 and 9×8, suggesting that each fact is represented separately in memory.

(iii) Multiplication problems with large operands were more error prone than those with small operands (problem size effect), although there was some inter-individual variability.

(iv) Finally, patients tended to produce erroneous results belonging to the multiplication table and, moreover, often falling within the same row or column as the correct result (e.g. $7 \times 8 = 48$).

Based on the performance of normal subjects in single-digit addition and multiplication, several models of arithmetic fact retrieval have been proposed (e.g., Ashcraft & Battaglia, 1978; Ashcraft, 1987; Campbell, 1987; Campbell & Oliphant, 1992; Siegler, 1988; for review see Ashcraft, 1992). McCloskey et al. (1991) have discussed how evidence from brain-damaged patients fits these various models. Early table-search models, based on a spreading of activation through a data structure analogous to a two-dimensional multiplication table (e.g. Ashcraft & Battaglia, 1978), did not seem readily compatible with the non-uniformity of calculation impairments. More recent models, based on network-like associations between the operands and the operation result, accommodate the data better. According to McCloskey and his colleagues, impairments of arithmetic fact retrieval are best modeled as a weakening of the associative connections between operand nodes and answer nodes. Upon presentation of the operands, the weakened connections do not reliably activate the correct result. Occasionally, an unrelated stored arithmetic fact (Ashcraft, 1987; Campbell & Graham, 1985) or a connexion linking the operands to an incorrect result (Siegler, 1988) are activated.

While these studies provide an adequate characterization of the cognitive deficit underlying amnesia for arithmetic facts, they generally do not consider the possible links between this impairment and other aphasic or amnesic difficulties outside the sphere of number processing. Although perfectly legitimate for testing various models of number processing, this approach leaves open the possibility that calculation impairments may reflect, at least in some cases, a more general deficit. This possibility may have important bearings on the issue of the relations between language and number processing systems (e.g. Campbell & Clark, 1986; Clark & Campbell, 1991). Furthermore, taking into account theoretical and experimental advances in the domain of memory or language may help elucidate the mechanisms underlying numerical deficits.

In the present study, we try to analyze a deficit of arithmetic fact retrieval in the light of concepts derived from studies of memory. Amnesic patients often show some degree of preservation of apparently inaccessible information. Adequate experimental procedures show that stored knowledge may influence the behavior of patients although this information cannot gain access to consciousness, nor be expressed verbally (Schacter, 1987). This preservation of *implicit* memory, as opposed to *explicit* memory, has been mainly demonstrated in amnesic patients. However this idea can be profitably extended to other domains such as language, vision, or number processing (Schacter, McAndrews, & Moscovitch, 1988). If deficits of arithmetic fact retrieval were due to an incomplete weakening of mnesic associations, rather than to a complete disruption of operand-to-answer connections, one might expect the weakened memories to have a covert influence on task performance even when they fail to promote explicit recall. It has also been shown that amnesic patients, as well as normal subjects to a lesser degree, perform much better in tests of explicit memory when submitted to *recognition* rather than to *recall* tasks (Delis, 1989). However, recognition memory has rarely been assessed in anarithmetia. McCloskey and his colleagues, for instance, had mainly asked patients to produce the result of operations (e.g. $3 \times 4 = ?$), which can be viewed as a typical recall task. Recognition tasks such as operation verification (e.g. $3 \times 4 = 13?$) have been used much more rarely (McCloskey, Caramazza, and Basili, 1985).

Here we describe a relatively pure case of amnesia for arithmetic facts, with essentially no other accompanying deficits of language and number processing. In order to evaluate the precise functional origin of the impairment, we resort to contrasted tasks assessing recall *vs.* recognition, as well as implicit *vs.* explicit access to arithmetic memory. We will then discuss to which degree of detail current network models of arithmetic knowledge are able to account for the observed pattern.

CASE REPORT

The patient was a 26-year-old, right-handed, French–Portuguese bilingual woman. She had lived in France until the age of 21 and had been educated in monolingual French schools. In particular she was taught elementary arithmetic in French. She had received college education in accountancy and had been an office worker for a few years. Before the onset of her neurological history, she was normally proficient in all usual aspects of number manipulation.

In December, 1990, a left parieto-temporal arteriovenous malformation was revealed by a subarachnoid hemorrhage. There were no neurological or neuropsychological sequelae. In the course of the following year, the patient underwent four artificial embolizations of feeding arteries. Following a subsequent attempt at surgery, she experienced right hemiparesis, right homonymous hemianopia, slight aphasic disorders, and reading and calculation difficulties.

The following neuropsychological study was carried out 6 months after the operation. At that time, neurological examination was normal aside from hemianopia. The main neuropsychological findings are summarized in Table 1. On the Wechsler Adult Intelligence Scale-Revised (WAIS-R), her verbal IQ was 77 and performance IQ was 75. She actually performed somewhat below the expected level for her age and education on nearly every subtest (vocabulary, 8; similarities, 7; arithmetic, 4; code, 4; block design, 7; picture arrangement, 8; picture completion, 5). On the Wechsler Memory Scale-Revised (WMS-R), her verbal index was 62 (2.6 *SD* below the mean), whereas her visual index of 98 was within normal bounds. Her performance was normal on the Wisconsin Card Sorting Test (6 correct criteria).

The patient submitted to a French version of the Boston Diagnostic Aphasia Examination (Goodglass & Kaplan, 1972; Mazaux & Orgogozo,

TABLE 1
Various Normative Data

Age: 26 years	
Digit span	
Forward	5
Backward	3
WAIS-R	
Verbal IQ	77
Performance IQ	75
WMS-R	
Verbal index	62
Visual index	98
Aphasia severity rating (BDAE)	5

1982). Aphasia had entirely receded during the course of the past 6 months, and her performance was flawless in most subtests. She was only mildly impaired in comprehension of complex ideational material (9/12), comprehension of complex written paragraphs (7/10), and fluency on controlled association (15 names of animals in 1 min). The patient was perfect on an additional test of picture naming (40/40). In the context of a deficit of memory for multiplication tables (see below), it is interesting to single out tests of verbal production from memory in which her performance was flawless. She was fast and accurate in reciting automatic series (days of the week, months of the year, counting verbally) and in completing familiar proverbs. She also easily produced antonyms or associates to common nouns and adjectives (e.g. white → black; 21/21 correct) (Table 2).

The patient was asked to read aloud three lists made up of a total of 136 words of varied frequency, imageability, and grammatical category, as well as 56 nonwords, each differing by one letter from a real word. She made 5.9% errors (8/136) with words and 25% errors (14/56) with nonwords, indicating a mild reading deficit. The vast majority of errors were mispronunciations of a single letter or phoneme.

The patient was tested with a screening battery for number-related abilities. Her digit span was 5 forward and 3 backward. She could count from 1 up to 20 and back. Number reading and writing on dictation were excellent in both arabic and verbal notation. The patient could flawlessly select the larger of two arabic numerals. She was presented visually with single-digit numerals. For each numeral, she was asked to decide as fast as possible whether it was larger or smaller than 5 and to respond by pressing one of two keys. Over a total of 160 trials, her mean RT was

TABLE 2
Specific Neuropsychological Tests

	Percent correct
Reading	
Words	94.1
Digits	94.1
Non-words	75.0
Proverbs completion	100
Recitation of automatic series	100
Production of antonyms and associates	100
Picture naming	100
Writing digits	99.1
Counting to 20, forward and backward	100
Timed number comparison (larger-smaller)	95.6
Parity judgment	100
Multi-digit calculation procedures	100

547 msec and her error rate 4.4% (a normal level of performance; cf. Moyer & Landauer, 1967), and she showed a normal distance effect (she was slower when the numbers were numerically close to the standard 5). She correctly classified 11 arabic numerals as odd or even. She could also judge the adequacy of a given number of items to a verbally proposed real-world situation (e.g., "Nine children in a school" → too small). By contrast, she showed major difficulties in solving simple addition and multiplication problems, suggesting a deficit of memory for arithmetic facts. With addition problems, she generally reached the correct result through finger counting. With multiplication problems, she often manifested complete ignorance of the correct result. However, her conceptual understanding of multiplication was intact, since when presented with a multiplication problem such as 5 × 5, she spontaneously sketched 5 rows of 5 dots and counted them by successive additions.

When presented with complex multidigit addition and multiplication problems, the patient showed preserved knowledge of calculation procedures. She correctly aligned the digits, processed them in the appropriate order, and dealt adequately with zero and carry operations. However she paused and very often erred whenever a specific arithmetic fact had to be retrieved from memory (e.g. 4 × 6). When the experimenter provided her with the missing results, she was able to integrate it appropriately into the calculation, and to complete multidigit addition and multiplication problems normally. The patient's performance therefore illustrates the known dissociability of memory for arithmetic facts from knowledge of calculation procedures (Caramazza & McCloskey, 1987; Temple, 1991).

In summary, on clinical testing the patient appeared to suffer from a rather selective deficit of memory for arithmetic facts. In the following section, this deficit is further explored in a series of formal experimental tasks, aiming at better documenting the functional locus of the deficit. We suggest that the deficit did not result from an impaired access to memory, but rather from a genuine loss of stored knowledge. We also demonstrate the preservation of some limited aspects of arithmetic knowledge, together with a severe impairment in activating, implicitly or explicitly, the exact connection between a given arithmetic problem and its result.

EXPERIMENTAL STUDY

Experiment 1: Addition and Multiplication Tables

Method. The first experiment aimed at quantifying the patient's knowledge of addition and multiplication tables. If the deficit occurs at the level of a central amodal store for arithmetic facts, performance should not depend on the particular modality used for input or output. In a first session, all possible addition and multiplication problems involving digits 1–9 were presented visually in two separate blocks, with 81 problems in each block.

The patient had to read aloud each problem and to provide verbally the appropriate result. In a second session, the same problems were read aloud to the patient, who was asked to write down the result followed by the two operands. This procedure enabled us to check that addition and multiplication errors could not be attributed to comprehension, production, or transcoding difficulties with the operands.

Results. Results are summarized in Table 3. Error rates did not differ significantly across the two sessions ($p > .10$). The patient tended to make more errors when problems were presented verbally than when they were presented visually, both with addition problems (11.1% [9/81] vs. 19.8% [16/81]) and with multiplication problems (33.3% [27/81] vs. 42.0% [34/81]). However this non-significant trend could be due to the higher demands of auditory presentation on short-term memory.

With addition problems, the patient made relatively few errors (25/162 = 15.4%) and she appeared to rely overtly or covertly on a back-up procedure of finger counting (see Case Report). Most of her addition errors (20/25 = 80%) were within 1 or 2 units of the correct result, which is consistent with an occasional slip in finger counting. Moreover there was no consistency across sessions in the specific problems that lead to errors (Fischer's exact test, $p = .43$). Again this is suggestive of random errors in the backup counting procedure, and precludes any further understanding of her memory difficulties for addition tables.

TABLE 3
Results of Experiments

	Percent errors	
Experiment 1: Production of arithmetical facts		
Addition problems	15.4	
Auditory stimuli		11.1
Visual stimuli		19.8
Multiplication problems	37.6	
Auditory stimuli		33.3
Visual stimuli		42.0
Experiment 2: Verification of multiplication problems		
Overall performance	27.8	
Canonical presentation		22.2
Non-canonical presentation		33.3
Experiment 3: Multiple choice of multiplication results		
Overall performance	18.8	
Within-table errors		18.8
Out-of-table errors		0.0
Numerically close errors		17.2
Numerically distant errors		1.6
Experiment 4: Membership in the multiplication table		
Overall performance	26.3	
Errors with within-table numbers		14.8
Errors with out-of-table members		29.8

With multiplication problems, the error rate was significantly higher than with addition problems (61/162 = 37.6%, χ^2 (1 *df*) = 20.5, $p <$ .0001), reflecting the absence of any back-up counting strategy when memory retrieval failed. Most errors (50/61 = 82.0%) were within-table errors, i.e. they corresponded to the result of another multiplication of two single digits (e.g. $4 \times 5 = 25$). Most of these (40/50 = 80.0%) were also within the correct row or column of the multiplication table. Only eight errors were out-of-table errors which did not correspond to a valid multiplication product (e.g. $6 \times 4 = 34$). On three problems, the patient did not produce any answer. Three additional observations were made. First, there was a significant correlation between the erroneous response and the correct result ($p < .0001$ in both sessions), indicating that responses were not selected randomly, but respected the gross magnitude of the problem. Second, the same problems consistently tended to yield errors in both sessions (Fischer's exact test, $p = .008$). The errors to a given problem were not always the same, however (e.g. $8 \times 4 = 34$, $8 \times 4 = 40$), suggesting that the impaired arithmetical facts had not been replaced by a fixed incorrect response. Finally, problems with larger operands produced significantly more errors (correlation of the error rate with the sum of the operands: Spearman's rank correlation = .57, $p = .04$).

Transcoding difficulties could not by themselves explain the patient's calculation errors. In the first session, she misread only 19 out of 324 digits (5.9%). In the second session, she made only three errors in writing down the 324 digits (0.9%). Although they indicate a mild arabic numeral reading deficit, these percentages are much lower than the error rates in addition and multiplication problems. Furthermore, reading errors and calculation errors did not necessarily affect the same problems. On some trials, the patient misread one of the operands but nevertheless produced the appropriate result (e.g. when presented with 3 + 4 she said "3 + 9 = 7"). Since the "4" must have been recognized correctly for the patient to compute the correct result 7, it is probable that her comprehension of arabic digits was intact, and that the occasional reading errors resulted from a marginal number production impairment, independent of her main deficit of memory for arithmetic facts.

Discussion. Experiment 1 confirmed that the patient suffered from a selective impairment of memory for arithmetic facts, which could not be accounted for solely by a comprehension or production deficit. Furthermore, the severity of the deficit was comparable whether the problems were presented visually or auditorily, suggesting that it originated from a common functional locus regardless of the input notation (McCloskey, 1992). Finally the patient's errors were not random, suggesting that some arithmetic knowledge was still available: errors respected the approximate magnitude of the result, and generally fell within the correct column or row of the multiplication table.

The unavailability of arithmetic facts in a recall task, such as the one used in this experiment, does not imply a permanent destruction of all or part of arithmetic memories. It may result, alternatively, from an impaired access to an otherwise intact memory store. Within the framework of associative models of arithmetic memory, a functional deficit may reflect either the loss or permanent degradation of specific connections between operand and answer nodes (degraded memory store), or a weakening or destabilisation of input operand activations (impaired access). Among the evidence collected so far, the consistency across testing sessions in the particular problems yielding incorrect responses is generally taken to support the degraded store hypothesis (Shallice, 1988). However consistency is a disputable criterion, since it could in principle be explained equally well by a stable and permanent damage to access mechanisms (Rapp & Caramazza, 1992). In Experiments 2 to 4, we therefore assessed the putative preservation of arithmetic memories using recognition tasks rather than recall tasks.

Experiment 2: Verification of Multiplications

Method. Experiment 2 assessed multiplication abilities in the absence of any explicit recall and production of a numerical result. The patient was simply asked to judge whether a proposed multiplication problem was true or false. The presentation of the problems was also systematically manipulated. There is a well-known effect of context on the performance of normal subjects in memory tasks: the reproduction of the context in which information was first acquired often improves explicit recall and recognition (e.g. Thomson & Tulving, 1970; Underwood & Humphreys, 1979). Remembering even depends on whether the subject is placed in the same physical environment during the learning and test sessions (e.g. Godden & Baddeley, 1975; Smith, 1979). Might contextual cues help the patient remember arithmetic facts? In a first session, 36 problems consisting of all ordered pairs of digits 2–9 were read aloud to the patient in canonical order. This is the standard order in which multiplication tables are taught at school: the smaller operand first, followed by the larger one and then the proposed result (e.g. 4 × 8 . . . 32). The problems were read aloud with the appropriate rhythm and prosody used in French schools. Half of the proposed results were correct, and the other half were out-of-table numbers within 1–5 units of the correct result (e.g. 2 × 9 . . . 17). In a second session, the same problems were read aloud in non-canonical order: the proposed result first, followed by the larger operand and then the smaller one (e.g. 17 . . . 9 × 2). If the patient suffered only from a disorder of access to an otherwise intact memory store, the more familiar canonical presentation of problems might be expected to improve her retrieval of memorized arithmetical facts.

Results and discussion. Results are summarized in Table 3. The patient's error rate did not differ significantly between the canonical and non-canonical conditions (22.2% [8/36] vs 33.3% [12/36]; χ^2 (1 *df*) = 1.11, p = .29). The overall error rate of 27.8% (20/72) was not different from that observed in Experiment 1 (p = .19). This confirms that the patient's mild production deficit could by no means account for her impaired arithmetic knowledge, since no overt production was required in the present experiment.

In normal subjects, operation verification (e.g. true/false judgment of 2 + 2 = 5) has been directly compared with explicit production of an operation result (e.g. 2 + 2 = ?). The two tasks were found to tap somewhat different mechanisms (Campbell, 1987; Zbrodoff & Logan, 1990). Verification of an operation may entail production of the result (2 + 2 = 4) and then comparison with the proposed result (4 ≠ 5). However, the entire equation "2 + 2 = 5" may also be compared against memory, and the result judged for familiarity and for plausibility, in an attempt to short-cut the lengthy computation of the exact result. In normal subjects and even in some acalculic patients, grossly false operations (e.g. 2 + 2 = 9), and operations that violate odd/even rules (e.g. 2 + 2 = 5), are rejected rapidly (e.g. Ashcraft & Stazyk, 1981; Dehaene & Cohen, 1991; Krueger & Hallford, 1984; Krueger, 1986). In the present experiment, all results were of a plausible magnitude. However the patient's performance did not vary whether the parity of the proposed result matched or did not match the parity of the actual result (respectively 30.8% vs. 43.5%). For instance she classified 2 × 2 = 5 as correct, in spite of the fact that 2 is even and 5 is odd. Thus, resorting to a recognition rather than to a recall task did not improve her performance.

Moreover, in normals, recognition improves when materials are presented in a familiar context as opposed to a novel context (e.g. Underwood & Humphreys, 1979), suggesting that contextual cues can facilitate memory retrieval (although to the best of our knowledge, this effect has not been studied in arithmetic). The absence of any difference between canonical and non-canonical presentations in our patient may be interpreted as an absence of context effects, and is therefore suggestive of a permanent loss of at least some arithmetic facts.

In Experiment 1, the patient produced only a few out-of-table errors. It was therefore unexpected that in the present task, 14 of 20 errors were failures to reject false multiplication problems with an out-of-table result. Differences in task demands may explain this discrepancy. In the present experiment, all proposed results were of a plausible magnitude, possibly biasing the patient toward responding that they were correct. The next experiment, in which we shall extend our exploration of the patient's recognition memory for arithmetic facts, was specifically designed to disentangle the effects of numerical proximity and table membership.

Experiment 3: Multiple-Choice Task

Method. Experiment 3 attempted to identify the factors that determined the patient's selection of a multiplication result. Sixty-four multiplication problems, consisting of all possible pairs of digits 2–9, were presented visually together with five proposed results. The patient was asked to select the correct one. In addition to the correct result, four foils were presented. Two were within the correct row or column of the multiplication table, one being numerically close to the correct result, and the other being more distant. The other two were out-of-table numbers, one close to and the other distant from the correct result. Thus the two factors of numerical proximity and table membership were crossed.

Results and discussion. Results are summarized in Table 3. The patient made 18.8% errors (12/64). This figure, while clearly abnormal, is nevertheless well above chance level. In accordance with the results of Experiment 1, all errors consisted in selecting a within-table result. In 11 out of 12 errors, the selected number was also numerically close to the correct result. These data confirm that even though a multiplication problem often did not activate its correct result in memory, the patient still knew the approximate magnitude of the result and also had some preserved knowledge of the set of within-table numbers. If the latter hypothesis is correct, she could be expected to correctly sort numbers according to their membership in the multiplication table. This was tested in the next experiment.

Experiment 4: Table Membership Judgment

Method. All numbers from 10 to 89 were read aloud to the patient in random order. She was asked to decide whether each number was a possible result for a multiplication of two single digits. She was encouraged to respond rapidly and without calculation, on the basis of familiarity. Numbers fell into 3 classes: 27 within-table numbers, 33 decomposable out-of-table numbers (e.g. 60, 77), and 20 prime numbers.

Results and discussion. Results are summarized in Table 3. The patient easily understood the task and responded rapidly and confidently. She made 26.3% errors (21/80), misclassifying 14.8% (4/27) of within-table numbers and 29.8% (17/57) of out-of-table numbers. It is noteworthy that there was no significant difference in error rates between decomposable and prime out-of-table numbers (36.4% [12/33] vs. 25.0% [5/20]; χ^2 (1 *df*) < 1). If the patient had based her judgment on an arithmetic evaluation of divisibility, she would have made a larger number of errors with decomposable numbers such as 82 or 34. That this was not the case suggests that she tried to use a more direct judgment of familiarity of the targets.

There was no obvious relation between errors in membership judgments and in explicit production of multiplication results. The patient correctly classified the numbers 27, 48, 49, and 72 as belonging to the multiplication table even though she had never produced them in response to any multiplication problem in Experiment 1 (e.g. to 3×9 she responded 16, 18, 28 or 64 but never 27). Conversely she misclassified the numbers 20, 24 and 81 as *not* belonging to the table even though she had produced them as multiplication responses in Experiment 1 (e.g. she *always* responded 81 to 9×9, 8×9, or 9×8). Finally she correctly classified 34 as not belonging to the table even though she had produced it erroneously in Experiment 1 in response to 4×7, 8×4, 9×3, 6×4, 4×6, and 3×7!

The clearest outcome of this experiment is that the patient's overall performance level was quite similar to that obtained in the above experiments, further confirming an impaired arithmetic memory affecting recognition as well as recall. Still, she was somehow able to recognize a number such as 64 as belonging to the multiplication table (85.2% correct), possibly because she could retrieve some combinations of operands leading to this result (perhaps including false ones such as "3 × 9 = 64"!). However she also often erred by erroneously classifying out-of-table numbers as within-table (29.8% "commission errors"), perhaps for the same reason that she could easily think of (incorrect) multiplications yielding them as a result. This strategy may have introduced an element of randomness in her performance, thereby explaining the discrepancy between her errors in membership judgements and in calculation tasks.

Experiments 1–4 demonstrated that the patient was equally impaired in recall and in recognition tasks of arithmetic memory. However, despite her severe inability to retrieve specific arithmetic facts, she still possessed some form of knowledge of multiplication facts, as evidenced by her preference for numerically close, within-table results. It has been repeatedly shown that on tests of implicit memory, amnesics may perform at a normal level despite being unable to consciously activate the relevant memories (e.g., Schacter *et al.*, 1988). Experiment 5 therefore aimed at probing the patient's implicit memory for multiplication facts.

Experiment 5: Timed Addition Verification

A number of experiments with normal subjects have evidenced an automatic activation of arithmetic facts that were not relevant to the requested task (e.g. LeFevre, Bisanz & Mrkonjic, 1988; Winkelman & Schmidt, 1974; Zbrodoff & Logan, 1986). Zbrodoff and Logan (1986) asked subjects to verify addition problems such as "3 + 4 = 12", in which the proposed false result was in fact the product of the two operands. Rejection of this type of problems proved specially difficult, suggesting that the subjects could not avoid accessing multiplication facts when only addition was required. We adapted this procedure in order to study whether the same phenomenon could be obtained with our patient.

Method. A list of 63 addition problems was presented visually on a computer screen in horizontal format (e.g. "4 + 9 = 36"). The patient was asked to respond as fast as possible by pressing the right-hand key if the result was correct, and the left-hand key if the result was false. The list included all possible ordered pairs of digits 2–9, except 2 + 2 for which both addition and multiplication yield the same result. One third of problems (21) were correct for addition. Another third were false but the proposed result was the product of the two operands. The remaining 21 problems were false, and the proposed result fell within 1 or 2 units of the product. Problem difficulty was roughly equated by matching the size of the operands across these three categories. The experimental list, preceded by four training trials, was presented twice to the patient as well as to six age-matched controls (mean age = 28 years, SD = 3.8).

Results and discussion. The patient made only six errors in this task (4.8%). An ANOVA was used to compare RTs for correct responses across the three problem types: correct, false with multiplication result, and false with control result. If the patient could activate some preserved, although consciously inaccessible, knowledge of multiplication facts, then one would expect longer latencies for rejecting addition problems with a multiplication result than for rejecting addition problems with a control result. However, as shown in Fig. 1 response times did not differ significantly between the two types of false problems ($F(1, 77) < 1$). If anything the patient was 57 msec *faster* in rejecting multiplication results (mean RT = 1403 msec) than in rejecting control results (mean RT = 1460 msec). She was significantly slower with correct addition problems (mean RT = 3578 msec, $p < .0001$). She appeared to have no immediate recognition of correct addition problems, and often had to count in order to verify them. Most incorrect problems, on the other hand, were so grossly false that she could reject them rapidly, without counting, and without even noticing the presence of multiplication results.

An analysis of the results of control subjects confirmed that the absence of an inhibition effect with multiplication results in the patient was not due to an inadequate experimental design. The mean RTs of the six control subjects were of 1057 msec in the multiplication condition (range:

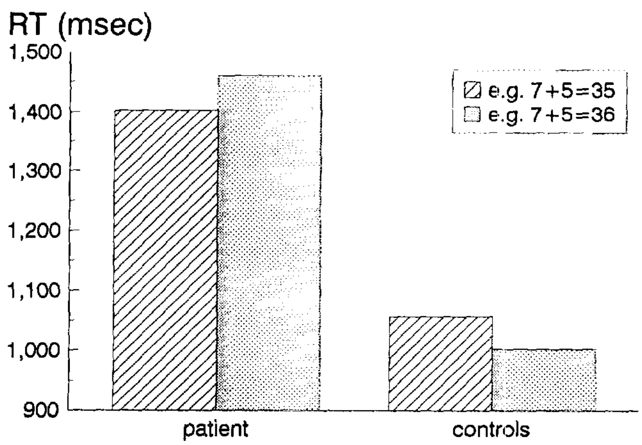

FIG. 1. Mean reaction times of the patient and control subjects in the timed addition verification task (Experiment 5). While controls were slower with addition problems whose proposed false result was the product of the two operands (hatched) than with control problems (grey), the patient showed no difference between these two conditions.

842 msec to 1410 msec), 1003 msec in the control condition (range: 806 to 1256 msec), and 1110 msec with exact problems (range: 809 to 1741 msec). The difference in rejection latencies between multiplication results and control results was marginally significant ($p = .06$), replicating Zbrodoff and Logan's (1986) finding of an increased difficulty in rejecting false addition problems with a multiplication result. The patient's results fell marginally off the distribution of difference scores of control subjects ($z = -1.61$, one-tailed $p = .054$). This suggested that the absence of an inhibition effect was unlikely to be found in a normal subject and might therefore be related to the patient's acalculia. In summary, no evidence of an intact implicit memory for arithmetic facts could be found in Experiment 5.

GENERAL DISCUSSION

Summary of Observed Dissociations

We have documented the case of a patient presenting with a selective impairment of memory for addition and multiplication tables. This deficit contrasted with a number of preserved abilities. First, aside from a moderate word reading deficit, the patient was not aphasic on standard neuropsychological batteries. In particular she was excellent in verbal tests superficially analogous to the production of memorized arithmetic facts, for instance proverbs completion, recitation of automatic series, and the production of associates and antonyms. Second, the patient was only very mildly impaired in transcoding tasks such as reading numerals or writing them to dictation. This contrasted sharply with the severity of her dyscalculia. Third, her deficit concerned only the memory for elementary arithmetic facts. Her conceptual understanding of numerical operations, as well as her knowledge of multidigit arithmetic procedures, were intact.

In the light of these dissociations, the term "pure amnesia for arithmetic facts" seems an appropriate description of the patient's deficit. The deficit was indeed relatively pure when contrasted against (1) preserved linguistic abilities, including language comprehension, production, and verbal long-term memory for proverbs and antonyms; (2) preserved number comprehension and transcoding; and (3) preserved knowledge of calculation procedures. This observation lends support to a modular view of number processing (e.g. McCloskey, 1992; Dehaene, 1992), according to which memory for arithmetic facts can be selectively impaired while leaving all other number processing abilities intact.

The Extent of Amnesia for Arithmetic Facts

Our tests were mainly directed at assessing the extent of the patient's loss of arithmetic facts. Amnesic patients often show a variable performance level depending on the nature of the memory task used. They may perform better in recognition than in spontaneous recall (Delis, 1989), and they often show a preservation of implicit memory even when explicit recall seems impossible (e.g. Schacter, 1987; Schacter *et al.*, 1988). However in the case of our patient, arithmetic knowledge seemed totally lost. Her deficit was equally severe on tasks assessing recall, recognition, or implicit access to arithmetic facts. She had similar difficulties recalling a multiplication result (Experiment 1), selecting the correct multiplication result among many (Experiment 2), recognizing if a proposed multiplication problem was correct or not (Experiment 3), or recognizing a number as a possible multiplication result (experiment 4).

In Experiment 2, using a more familiar canonical presentation of problems did not facilitate the recognition of arithmetic facts. Recognition of correct multiplication problems, even when presented in the familiar wording used for drilling arithmetic at school, was not better that recall. Furthermore, Experiment 5 demonstrated the absence of an automatic activation of multiplication facts in the patient, in a situation where normal subjects show an irrepressible implicit activation. The observation of a deficit on many tests of arithmetic memory supports the hypothesis of a permanent loss of arithmetic memory, rather than a variable or fleeting deficit of access or of explicit recall.

The criterion of error consistency is also considered useful to separate access deficits from degraded store disorders (Warrington & Shallice, 1979; Shallice, 1988, pp. 282–283). Across the two testing sessions of Experiment 1, we found a significant consistency in the particular multiplication problems which yielded incorrect responses. This is in agreement with a permanent loss of the corresponding facts, although it might perhaps also be explained as a permanent access deficit (for discussion see Rapp & Caramazza, 1992).

It is worth noting that, contrary to the present case, some patients with impaired memory for arithmetic facts are best described as suffering from a variable access deficit. Patient D.R.C., studied by Warrington (1982), experienced occasional difficulties in retrieving the result of addition and multiplication problems. However in contrast with the present case, D.R.C.'s performance was highly inconsistent from trial to trial. The very same problem could be solved instantaneously on one trial and require lengthy counting on another trial. On this basis, Warrington (1982) argued that "inaccessibility rather than damage to the semantic entries of arithmetic facts [was] at the core of [D.R.C.'s] acalculia."

McCloskey et al. (1991) have described several patients with impaired memory for arithmetic facts. Each patient was repeatedly tested on all single-digit multiplication problems from 0×0 to 9×9. Although the authors did not specifically address this issue, it is possible to reanalyse the tables of data that they provide in the context of impaired access vs.

degraded store deficits. For instance, patient I.E. showed an error pattern suggestive of a degraded store disorder similar to our patient. Her error rates to individual problems tended to follow a bimodal distribution with one peak close to 0% errors and a second peak close to 100% errors (see Table 7 in McCloskey et al., 1991). This suggests that a given problem was either known and accessible, or was lost from memory. By contrast, most of the other patients did not show such consistency across problems. Their error rates ranged continuously from 0 to 100%, suggesting that a given problem could be solved on one occasion but not on another. Taken together, these observations illustrate the possibility of a double dissociation, within the arithmetic system, between the memory store for arithmetic facts and the mechanisms that access it.

Preserved Aspects of the Patient's Performance

The pattern of the patient's errors indicated that her performance in multiplication was not random. First, in multiplication, she generally selected a response that fell within the table of possible multiplication results (within-table numbers), and often within the correct row or column of this table. Second, the number that she selected was always numerically close to the correct result. These phenomena were observed in spontaneous production as well as in a multiple-choice task. Similar patterns of errors have been reported in previous studies of acalculic patients (McCloskey, Aliminosa & Sokol, 1991; Sokol et al., 1991) and can be experimentally elicited in normal subjects (e.g., Campbell & Graham, 1985; Stazyk, Ashcraft & Hamann, 1982).

McCloskey et al. (1991) have provided a detailed theoretical discussion of acalculic patients' multiplication errors, which need not be reiterated here. Our observations are compatible with their conclusion that deficits of arithmetic memory are due to the permanent degradation of some memory connections linking a representation of the operands with a representation of the operation result. Associations between operands and answers are postulated in all current models of arithmetic memory (e.g. Ashcraft, 1987; Campbell, 1987; Campbell & Oliphant, 1992; Siegler, 1988). It is assumed that the degradation of some of these associations disrupts the flow of processing in memory, and leads to the activation of an incorrect output answer node. This accounts for the fact that most errors fall within the set of possible multiplication results. The additional postulate of a local spread of activation among operand nodes (e.g. Ashcraft, 1987), or of an additional activation of magnitude information (e.g. Campbell & Oliphant, 1992) may suffice to explain why errors often correspond to a neighboring fact belonging to the correct row or column of the multiplication table (e.g. $6 \times 7 = 48$).

In this hypothesis, deficits of arithmetic memory are ascribed to a partial disconnection of some memorized associations within the memory store. In the framework of network models of arithmetic memory, one may also question whether the operand nodes and the answer nodes, which constitute the inputs and outputs of this memory system, were also lesioned. In the present case, perception and comprehension of arabic digits were demonstrably intact, as attested by the patient's good performance in number comparison and in transcoding tasks. Thus the operands of an operation such as "3 × 4 = ?" were presumably adequately perceived and used for accessing arithmetic memory. However, it seems likely that at least some "answer nodes" were lesioned. First, some possible products (e.g. 27) were never produced in response to any multiplication problem and were misclassified in the table-membership task. Second, some numbers that are not possible products (e.g. 34) were occasionally produced in response to a multiplication problem or accepted as correct in multiplication verification. The disturbance of "answer nodes" was probably moderate, as demonstrated by (i) the fact that the patient almost exclusively produced or selected plausible multiplication results in Experiments 1 and 3 and (ii) her above-chance performance in assessing whether a two-digit number belonged to the multiplication table or not (Experiment 4). In summary, current models of arithmetic can adequately explain the patient's acalculic deficit, at a functional level, as a specific loss of some connections and output nodes within the arithmetic memory store.

REFERENCES

Ashcraft, M. H., & Stazyk, E. H. 1981. Mental addition: A test of three verification models. *Memory And Cognition*, **9**, 2, 185–196.

Ashcraft, M. H., & Battaglia, J. 1978. Cognitive arithmetics: evidence for retrieval and decision processes in mental addition. *Journal of Experimental Psychology: Human Learning and Memory*, **4**, 5, 527–538.

Ashcraft, M. H. 1987. Children's knowledge of simple arithmetic: A developmental model and simulation. In J. Bisanz, C. J. Brainerd, and R. Kail (Eds.), *Formal methods in developmental psychology*. Berlin: Springer. Pp. 302–338.

Ashcraft, M. H. 1992. Cognitive arithmetic: A review of data and theory. *Cognition*, **44**, 75–106.

Campbell, J. I. D. 1987. Network interference and mental multiplication. *Journal of Experimental Psychology: Learning, Memory and Cognition*, **13**, 1, 109–123.

Campbell, J. I. D., & Clark, J. M. 1988. An encoding complex view of cognitive number processing: Comment on McCloskey, Sokol, & Goodman (1986). *Journal of Experimental Psychology: General*, **117**, 204–214.

Campbell, J. I. D., & Graham, D. J. 1985. Mental multiplication skill: Structure, process, and acquisition. *Canadian Journal of Psychology*, **39**, 338–366.

Campbell, J. I. D., & Oliphant, M. 1992. Representation and retrieval of arithmetic facts: A network-interference model and simulation. In J. I. D. Campbell (Ed.), *The nature and origin of mathematical skills*. Amsterdam: Elsevier.

Caramazza, A., & McCloskey, M. 1987. Dissociations of calculation processes. In G. Deloche & X. Seron (Eds.), *Mathematical disabilities: A cognitive neuropsychological perspective*. Hillsdale, NJ: Erlbaum. Pp. 221–234.

Clark, J. M., & Campbell, J. I. D. 1991. Integrated versus modular theories of number skills and acalculia. *Brain and Cognition*, **17**, 204–239.

Dehaene, S., & Cohen, L. 1991. Two mental calculation systems: A case study of severe acalculia with preserved approximation. *Neuropsychologia* **29**, 1045–1074.

Dehaene, S. 1992. Varieties of numerical abilities. *Cognition*, **44**, 1–42.

Delis, D. C. 1989. Neuropsychological assessment of learning and memory. In F. Boller & J. Grafman (Eds.), *Handbook of neuropsychology*. Amsterdam: Elsevier. Vol. 3, pp. 3–33.

Godden, D., & Baddeley, A. D. 1975. Context-dependent memory in two natural environments. *British Journal of Psychology*, **66**, 325–331.

Goodglass, H., & Kaplan, E. 1972. *The Boston diagnostic aphasia examination*. Philadelphia: Lea & Febiger.

Hécaen, H., Angelergues, R., & Houillier, S. 1961. Les variétés cliniques des acalculies au cours des lésions rétro-rolandiques: Approche statistique du problème. *Revue neurologique*, **105**, 85–103.

Henschen, S. E. 1919. Uber Sprach-, Musik- und Rechenmechanismen und ihre Lokalisationen im Grosshirn. *Zeitschrift fur die gesamte Neurologie and Psychiatrie*, **52**, 273–298.

Krueger, L. E., & Hallford, E. W. 1984. Why 2 + 2 = 5 looks so wrong: On the odd-even rule in sum verification. *Memory & Cognition*, **12**, 171–180.

Krueger, L. E. 1986. Why 2 × 2 = 5 looks so wrong: On the odd-even rule in product verification. *Memory & Cognition*, **14**, 141–149.

LeFevre, J., Bisanz, J., & Mrkonjic, L. 1988. Cognitive arithmetic: Evidence for obligatory activation of arithmetic facts. *Memory & Cognition*, **16**, 45–53.

Mazaux, J. M., & Orgogozo, J. M. 1982. *Echelle d'évaluation de l'aphasie: Adaptation française du Boston diagnostic aphasia examination*. Issy-les-Moulineaux: Editions Scientifiques et Psychologiques.

McCloskey, M., Caramazza, A., & Basili, A. G. 1985. Cognitive mechanisms in number processing and calculation: Evidence from dyscalculia. *Brain and Cognition*, **4**, 171–196.

McCloskey, M., Aliminosa, D., & Sokol, S. M. 1991. Facts, rules, and procedures in normal calculation: Evidence from multiple single-patient studies of impaired arithmetic fact retrieval. *Brain & Cognition*, **17**, 154–203.

McCloskey, M., Harley, W., and Sokol, S. M. 1991. Models of arithmetic fact retrieval: An evaluation in light of findings from normal and brain-damaged subjects. *Journal of Experimental Psychology: Learning, Memory and Cognition*, **17**, 377–397.

McCloskey, M. 1992. Cognitive mechanisms in numerical processing: Evidence from acquired dyscalculia. *Cognition*, **44**, 107–157.

Moyer, R. S., & Landauer, T. K. 1967. Time required for judgements of numerical inequality. *Nature*, **215**, 1519–1520.

Rapp, B., & Caramazza, A. 1992. On the distinction between deficits of access and deficits of storage: A question of theory. In A. Caramazza and H. Whittaker (Eds.), *Theoretical and methodological issues in cognitive neuropsychology*, in press.

Schacter, D. L., McAndrews, M. P., & Moscovitch, M. 1988. Access to consciousness: dissociations between implicit and explicit knowledge in neuropsychological syndromes. In. L. Weiskrantz (Ed.), *Thought without language*. New York: Oxford University Press. Pp. 242–277.

Schacter, D. L. 1987. Implicit memory: History and current status. *Journal of Experimental Psychology: Learning, Memory and Cognition*, **13**, 501–518.

Shallice, T. 1988. *From neuropsychology to mental structure*. Cambridge University Press.

Siegler, R. S. 1988. Strategy choice procedures and the development of multiplication skill. *Journal of Experimental Psychology: General*, **3**, 258–275.

Smith, S. M. 1979. Remembering in and out of context. *Journal of Experimental Psychology: Human Learning and Memory*, **5**, 115–124.

Sokol, S. M., McCloskey, M., Cohen, N. J., & Aliminosa, D. 1991. Cognitive representations and processes in arithmetic: Inferences from the performance of brain-damaged subjects. *Journal of Experiment Psychology: Learning, Memory, and Cognition*, **17**, 355–376.

Stazyk, E. H., Ashcraft, M. H., & Hamann, M. S. 1982. A network approach to mental multiplication. *Journal of Experimental Psychology: Learning, Memory and Cognition*, **8**, 320–335.

Temple, C. M. 1991. Procedural dyscalculia and number fact dyscalculia: Double dissociation in developmental dyscalculia. *Cognitive Neuropsychology*, **8**, 155–176.

Thomson, D. M., & Tulving, E. 1970. Associative encoding and retrieval: weak and strong cues. *Journal of Experimental Psychology*, **86**, 255–262.

Underwood, B. J., & Humphreys, M. 1979. Context change and the role of meaning in word recognition. *American Journal of Psychology*, **92**, 577–609.

Warrington, E. K., & Shallice, T. 1979. Semantic access dyslexia. *Brain*, **102**, 43–63.

Warrington, E. K. (1982). The fractionation of arithmetic skills: A single case study. *Quarterly Journal of Experimental Psychology*, **34A**, 31–51.

Winkelman, H. J., & Schmidt, J. 1974. Associative confusions in mental arithmetic. *Journal of Experimental Psychology*, **102**, 734–736.

Zbrodoff, N. J., & Logan, G. D. 1986. On the autonomy of mental processes: A case study of arithmetic. *Journal of Experimental Psychology: General*, **115**, 118–130.

Zbrodoff, N. J., & Logan, G. D. 1990. On the relation between production and verification tasks in the psychology of simple arithmetic. *Journal of Experimental Psychology: Learning, Memory and Cognition*, **16**, 83–97.