CHAPTER 22

Levels of Representation in Number Processing

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Numbers can be manipulated mentally in several formats such as Arabic notation, spelled-out numerals, and an abstract quantity representation. Neuropsychological models of number processing have attempted to specify the architecture in which mental representations of numbers are interconnected. McCloskey and his colleagues initially proposed a simple model with a single central abstract quantity representation interfaced by notation-specific input and output modules. Although this model sucessfully accounts for several of the peripheral deficits of number processing, difficulties arise with the hypothesis of an obligatory step of semantic interpretation of numbers. Several problematic cases suggest that there must be direct asemantic processing routes parallel to the semantic processing of numerical quantities. An alternative model, Dehaene and Cohen's triple-code model, describes both the functional architecture and the neural substrates of number processing. It successfully accounts for many types of numerical deficits, including the peculiar dissociations found in pure alexia, in callosotomy cases, in Gerstmann's syndrome, and in subcortical acalculias.

There is a domain of language in which we are all, in some sense, bilinguals: the domain of numbers. Like any other category of words, numbers can be spoken (/four/, /forty/) or spelled out (FOUR, FORTY). However, we are all familiar with a

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third symbolic notation, Arabic numerals (4, 40). Furthermore, numbers can also be conveyed in nonsymbolic ways such as sets of dots (::).

The human brain must contain mental representations and processes for recognizing, understanding, and producing these various notations of numbers and for translating between them. Hence, the number domain provides a manageably restricted area within which to study the representation of symbolic information in the human brain and the interplay between verbal and nonverbal formats of representation. It is hoped that the lessons that numbers might teach us about the organization of symbol systems in the human brain will turn out to be generalizable to other linguistic domains.

In the past decade, much progress has been made toward understanding the internal organization of the mental representations of numbers, their interconnections, and their neural substrates. In this chapter, we first describe some of the evidence for a quantity representation in humans, and the central role that is attributed to this quantity representation in McCloskey's modular model of number processing. We then discuss several single-case studies of patients with number-processing deficits, some that support McCloskey's assumptions and others that seem to invalidate it. We close by briefly presenting the triple-code model and how it accounts for the majority of these cases.

22-1. THE QUANTITY REPRESENTATION

Moyer and Landauer's (1967) seminal study of number comparison provided the first strong evidence for a quantitative representation of numbers in human adults. Measuring the time that it takes to select the larger of two digits, they found a *distance effect:* number comparison becomes systematically slower as the distance between the two numbers decreases. It is easier to decide, say, that 9 is larger than 5 than to decide that 9 is larger than 8. The effect was later extended to two-digit numerals (Dehaene, Dupoux, & Mehler, 1990). The reaction time curve for deciding whether a two-digit number is larger or smaller than 65 is remarkably smooth and shows no significant discontinuities at decade boundaries (Figure 1). Reaction times are influenced by the ones digits even though the tens digit is sufficient to respond: subjects respond "smaller" more slowly to 59 than to 51, although the 5 in the decades position readily indicates that both of these numbers are smaller than 65.

These results suggest that subjects do not compare numbers digit by digit. Rather, it was hypothesized that subjects mentally convert the target Arabic numeral into a continuous quantity, which they then compare to the reference quantity using a psychophysical procedure similar to the one used for comparing line lengths, weights, or other physical quantities. Several models of number comparison suppose that the human brain incorporates an *analogical representation of numerical quantities* that may be likened to a number line. In this representation, numbers are not represented by discrete symbols such as digits or words, but by distributions of activation whose overlap indicates how similar the quantities are. Several experiments with normal

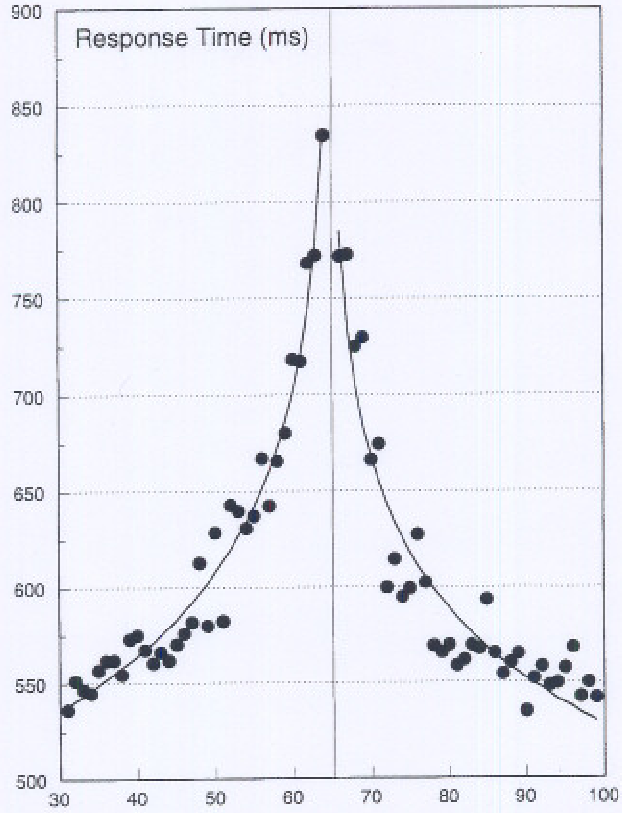

FIGURE 1 Distance effect in number comparison (data from Dehaene, Dupoux, & Mehler, 1990).

subjects confirm that the conversion from digits or number words to the corresponding quantities is fast and automatic and even occurs unbeknownst to the subject (review in Dehaene & Akhavein, 1995).

22-2. McCLOSKEY'S MODEL

What is the neuropsychological architecture in which Arabic, verbal, and quantity representations of numbers are embedded? McCloskey and his colleagues (McCloskey, Caramazza, & Basili, 1985; McCloskey, 1992) proposed a simple modular model that was to become, for a decade, the reference model for studies of number processing. The model places the quantity representation at a central point in processing and assumes that it is interfaced by specialized input and output modules. On the input side, a panoply of modules serves to convert Arabic digits and written or spoken numerals into internal quantities. On the output side, conversely, other modules are called for to write down Arabic digits and number words or to say numerals aloud. The most important assumption of the model is that each mental operation on numbers involves a conversion to the abstract semantic quantity representation. Arithmetic fact retrieval and calculation procedures, in particular, are thought to operate exclusively on the quantity format (Figure 2).

McCloskey's modular stance was remarkably successful in classifying and in interpreting neuropsychological impairments of number processing as well as in discovering new dissociations. One of the first cases to be explored was patient H.Y. (McCloskey, Sokol, & Goodman, 1986). H.Y. made errors in reading Arabic numerals aloud—for instance, reading 5 as *seven* or 29 as *forty-nine*. A careful analysis indicated that the deficit was highly specific and modular. First, the patient still understood the Arabic numerals that he failed to read, as attested by his ability to compare them, to select a number of chips corresponding to an Arabic digit, to match an Arabic digit to a written word, or to verify written calculations. According to the logic of the model, this proved that the conversion of numerals into quantities had to be intact. Hence, the number reading deficit had to originate from the output side, in the process of converting quantities into spoken numerals.

Second, McCloskey and his colleagues showed that writing down Arabic digits in response to various tasks was fairly intact (2-4% errors), while speaking aloud the numerical answers was significantly more impaired (8-14% errors). For instance, when asked the number of eggs in a dozen, the patient wrote 12 while saying *sixteen*. This suggested a rather selective deficit of the spoken production module.

Third, a careful analysis of H.Y.'s reading errors indicated that they mostly constituted substitutions of one word for another, while the grammatical structure of hundreds, decades, and units was well preserved (e.g., stimulus 902, response *nine hundred six*). Furthermore, ones words were almost always replaced by other ones words,

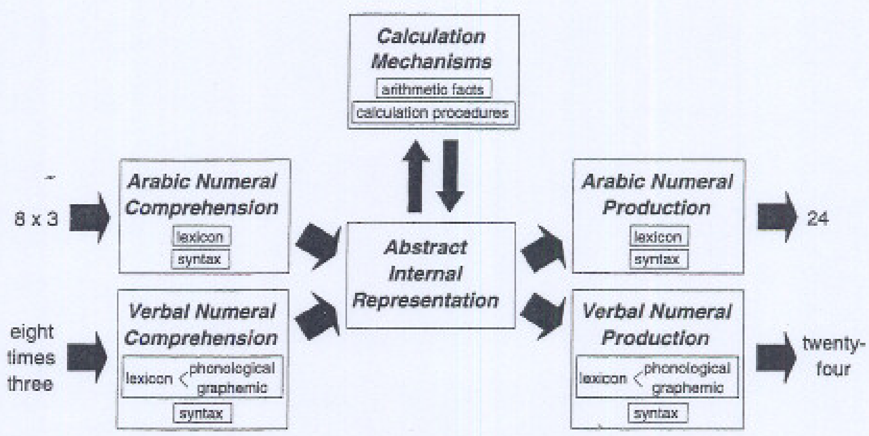

FIGURE 2 Schematic functional architecture of McCloskey's model.

teens words by other teens words, and tens words by other tens words. In the final analysis, the deficit was pinpointed to an impairment in using quantity information to select the phonological form of the word in lexical stacks for ones, teens, and tens words. When reading 15, the patient prepared to read aloud the fifth element of the teens category (eleven, twelve, thirteen, fourteen, *fifteen*), but he mistakenly selected, say, the eighth element *eighteen* instead. Although some details of the word substitutions remained unexplained (see Campbell & Clark, 1988; Sokol, Goodman-Schulman, & McCloskey, 1989), the bulk of the deficit could be reduced to this highly restricted lesion of the modular model.

McCloskey's model successfully predicted several other types of number processing deficits (reviews in McCloskey, 1992; McCloskey, & Caramazza, 1987; Caramazza & McCloskey, 1987). In number reading, patients similar to H.Y. were found who selected words in the wrong numerical category but at the correct ordinal location in the category (e.g., reading 2 as *twelve* or 15 as *fifty*). Others had preserved selection procedures for single words, but generated an incorrect syntactic sequence of words when producing complex numerals (e.g., reading 218 as *two thousand one hundred and eight*).

Similar modular deficits and dissociations were found in calculation. Some patients' only deficit was in identifying the operation signs +, ×, and so on. For others, the deficit was limited to arithmetic fact retrieval—they failed to retrieve single facts such as 2 × 3 or 5 + 5 in memory, but could otherwise execute the procedures for multidigit addition and multiplication to near perfection. And conversely, other patients suffered from a selective impairment of calculation procedures, while their memory for individual facts was intact. Hence there was a double dissociation between facts and procedures in calculation, compatible with McCloskey's hypothesis that these correspond to different number-processing modules.

Elegant support for the model was also accrued when Macaruso, McCloskey, and Aliminosa (1993) showed that they could account not only for the qualitative dissociations of patients with highly selective deficits, but also for the quantitative error rates of a patient with multiple deficits. Patient R.H. exhibited mild to severe deficits in several number-transcoding tasks. For instance, he made 25% errors in reading Arabic numerals aloud, 21% errors in writing Arabic numerals to dictation, and 88% errors in converting Arabic numerals to their spelled-out form. These deficits could clearly not be accounted for by a single lesion site in the model. However, Macaruso et al. (1993) found that the patient's quantitative error rates in transcoding back and forth from Arabic, spelled-out, spoken, and dot notations of numbers could be explained by assigning probabilities of errors to each component of the modular model. For instance, the 25% error rate in Arabic numeral reading resulted from a 3% error rate in the Arabic identification module and a 22% error rate in the spoken production module. The error types also conformed to this "multiple-deficits" analysis. For instance, the patient made word substitution errors whenever the task required spoken production of numbers, whether the input was in Arabic, in word, or in dot format—a finding compatible with the existence of a lesioned "number production module" shared between all these tasks.

22-3. SOME PROBLEMATIC CASES

In spite of these successes, McCloskey's model still generates much controversy. In particular, the hypothesis of a central semantic representation of numbers, accessed in all tasks involving numbers, is hotly debated. To take an extreme example, the model predicts that the mere repetition of a spoken number word necessarily involves going through the semantic representation—an implausible prediction, as acknowledged by McCloskey himself (McCloskey, 1992). The absence of a direct surface transcoding route, for instance, going directly from the orthographic form of a number word to its pronunciation, conflicts with neuropsychological evidence from language impairments outside the numerical domain.

A first specific case that was found difficult to explain within McCloskey's framework was Cohen and Dehaene's (1991) patient Y.M. This patient showed a reading impairment characterized by digit substitutions (3 was read *eight*), an effect of visual similarity on errors (3 was rarely substituted with a visually dissimilar digit such as 1), and a spatial gradient of errors (errors mostly affected the leftmost digits of multidigit numerals). In McCloskey's framework, the fact that visual variables affected reading suggested a deficit within the visual Arabic identification module. Yet several observations conflicted with this hypothesis. Most notably, the patient was perfect in selecting the larger of two Arabic digits, and also surprisingly good in verifying additions such as $2 + 2 = 9$. The data, while not fully conclusive, suggested that the semantic route from Arabic digits to quantities was intact, while the route from Arabic digits to spoken words was impaired—a pattern forbidden in McCloskey's model.

Multiple routes also seemed necessary to explain a case reported by Cipolotti and Butterworth (1995). Their patient S.A.M. made frequent errors both when reading aloud Arabic and spelled-out numerals and when writing the same numbers down to dictation. Yet he was remarkably accurate in calculation tasks involving multidigit addition, subtraction, and multiplication problems, including tasks that required him to say the very same numbers that he had failed to read aloud! For instance, S.A.M. could say the result of $396 + 837$ without error, although he made errors in reading these numbers aloud. This striking dissociation suggests that the ability to produce Arabic or verbal numerals can be specific to particular task demands. Cipolotti and Butterworth (1995) suggest that reading aloud and writing down numbers to dictation makes use of direct asemantic routes different from those used in a calculation context.

Perhaps the clearest incompatibility with McCloskey's model came from a study of two patients with pure alexia (Cohen & Dehaene, 1995). Patients G.O.D. and S.M.A. suffered from highly similar infarcts of the left ventral occipito-temporal area, a region involved in high-level visual identification. Both were totally unable to read words. Arabic numerals were slightly better preserved: multidigit numerals were misread on 60-80% of trials and single digits on 8-18% of trials. Calculation on written operands was also impaired. For instance, when presented with $2 + 3$, the patients might say *seven*. That these calculation errors were due to a misidentification of the digits was shown by *(a)* the patients' perfect ability to perform the same calculation with spoken operands, and *(b)* the patients' reading errors in calculation: for instance, 2 + 3 was read *two plus five* and then solved as *seven,* indicating that the patients correctly computed the sum of the operands that they had misread.

So far, this deficit might be understood as a selective impairment of the Arabic number identification module. Contradicting this hypothesis, however, was the fact that both patients compared Arabic numerals with remarkable accuracy. When presented with 44 pairs of two-digit numerals, for instance, neither of them made any errors in pointing to the larger number, whereas they made, respectively, 89% and 91% errors in reading the very same pairs aloud. Some of the reading errors inverted the order of the numbers. For instance, 78 76 was read as *seventy eight, seventy nine*—yet the patient correctly pointed to 78 as the larger number.

This pattern of dissociation is clearly inconsistent with McCloskey's model. In this model, the preserved comparison of Arabic numerals should imply that the Arabic number identification module is intact. Similarly, the perfect performance with spoken inputs and outputs should mean that the verbal number comprehension, production, and calculation modules are intact. With all these intact components, the modular model should predict the reading aloud of Arabic numerals to be perfectly preserved—yet it was severely impaired. Cohen and Dehaene (1995) argued that these dissociations could only be explained by postulating two Arabic numeral identification systems, one involved in identification for the purpose of naming (which is impaired in pure alexia), and the other involved in identification for the purpose of accessing the quantitative semantic representation (which is preserved; more on this later).

A functionally similar case was presented by McNeil and Warrington (1994). Their patient H.A.R. also suffered from alexia without agraphia (although he was also aphasic and suffered from naming difficulties). Just like patients G.O.D. and S.M.A., he experienced severe difficulties in reading Arabic numerals aloud or in calculating with them. Yet he could calculate with near perfection with spoken operands and he could also compare Arabic numerals. An additional twist was that when calculating with written Arabic digits, only additions and multiplications were severely impaired: the patient was excellent in subtracting two digits that he failed to read aloud. It seems that the same procedure that enabled him to convert Arabic numerals into quantities during number comparison could also be used for subtraction—but not for addition or multiplication. According to McCloskey's model, such an operation-specific *and* number-specific deficit should not exist.

22-4. THE TRIPLE-CODE MODEL

The triple-code model was introduced by Dehaene (1992; Dehaene & Cohen, 1995) in an effort to account for these peculiar cases. The model postulates three main representations of numbers (Figure 3):

1. A visual Arabic code, localized to the left and right inferior ventral occipito-temporal areas, and in which numbers are represented as identified strings of digits. This representation, which we call the visual Arabic number form by

analogy with Shallice's visual word form, subserves multidigit operations and parity judgments (e.g., knowing that 12 is even because the ones digit is a 2).

2. An analogical quantity or magnitude code, subserved by the left and right inferior parietal areas, and in which numbers are represented as points on an oriented number line. This representation subserves semantic knowledge about numerical quantities, including proximity (e.g., 9 close to 10) and larger–smaller relations (e.g., 9 smaller than 10).
3. A verbal code, subserved by left-hemispheric perisylvian language areas, in which numbers are represented as a parsed sequence of words. This representation is the primary code for accessing a rote verbal memory of arithmetic facts (e.g., "nine times nine, eighty-one").

Figure 3 shows the patterns of interconnections that are postulated in the model. In the left hemisphere, all three cardinal representations (Arabic, verbal, and quantity) are interconnected by bidirectional translation routes, including a direct asemantic route for transcoding between the Arabic and verbal representations. In the right hemisphere, there are similar routes for translating back and forth between Arabic and quantity representations, but there is no verbal representation of numbers. Finally, it is assumed that the homologous Arabic and quantity representations in the two hemispheres are interconnected by direct transcallosal pathways.

How does the model account for pure alexia patients, who were found so problematic for McCloskey's model? Their lesion affected the left ventral occipito-temporal area and therefore predictably impaired the left visual word and number identification system. Hence, left-hemispheric verbal areas could not be directly informed about the identity of the visual stimulus, resulting in a severe impairment of reading aloud. Since the memory retrieval of rote arithmetic facts is assumed to depend on the verbal format, this reading deficit entailed a calculation deficit—the patients could not turn 3×3 into the verbal format *three times three,* which was necessary for them to retrieve the result *nine.* Yet the verbal circuit itself was intact, only deprived of visual inputs. Hence the patients could still calculate when an arithmetic problem was read

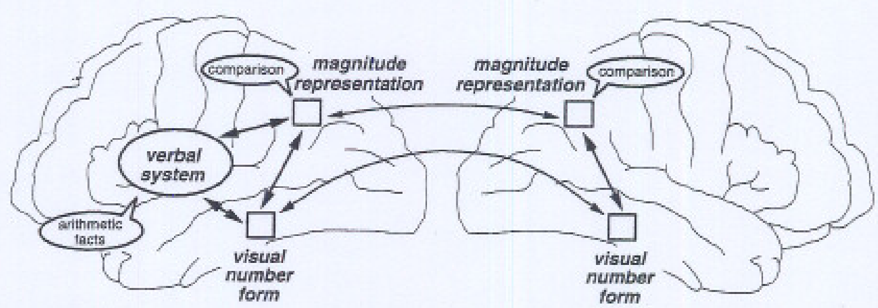

FIGURE 3 Schematic anatomical and functional architecture of Dehaene and Cohen's (1995) triple-code model.

aloud to them. And finally, their right hemisphere was fully intact, including its visual identification and magnitude representation areas. According to the triple-code model, these preserved right-hemispheric circuits underlie the patients' ability to compare the magnitudes of Arabic numerals that they fail to read (Cohen & Dehaene, 1995).

Very similar interpretations account for the above-described patients Y.M. (Cohen & Dehaene, 1991) and H.A.R. (McNeil & Warrington, 1994). Although we know very little about how mental subtraction is performed, clearly we do not learn subtraction tables by rote as we do for addition and multiplication. Dehaene and Cohen (1995) speculate that subtraction is more similar to number comparison and involves quantitative manipulations rather than rote verbal retrieval. This may explain why patient H.A.R. could still subtract Arabic numerals, while his inability to read them aloud severely affected his ability to add or multiply them.

A strength of the triple-code model is that it specifies cerebral localizations, however coarse, for the various representations of numbers. As a result, the model can predict the effect of specific anatomical lesions on number processing. A case in point is the split-brain syndrome. When the corpus callosum is severed, the model predicts that the left hemisphere should remain able to perform all sorts of calculations, because it contains the three cardinal representations of numbers (Arabic, verbal, and quantity). The right hemisphere, however, should be able to recognize Arabic numerals, to retrieve the quantity that they represent, but not to read aloud nor to perform calculations dependent on the verbal code. This predicted pattern of results is exactly what is found, both in the classical literature on split brains (Gazzaniga & Hillyard, 1971; Gazzaniga & Smylie, 1984; Seymour, Reuter-Lorenz, & Gazzaniga, 1994) and in a single-case study of number processing following an infarct of the posterior half of the corpus callosum (Cohen & Dehaene, 1996). When digits 5 and 6 are flashed in the left hemifield, the patients' right hemisphere can decide that 5 is smaller than 6, but it cannot read the digits aloud nor calculate 5 + 6. Similar results suggesting a right-hemispheric ability restricted to quantitative processing have been obtained in patients with extended left-hemispheric lesions (Dehaene & Cohen, 1991; Grafman, Kampen, Rosenberg, Salazar, & Boller, 1989). Patient N.A.U., for instance, could not tell whether 2 + 2 was 3, 4, or 5—but he knew that it had to be smaller than 9.

The triple-code model also accounts for some of the various types of acalculias and their anatomical correlates. According to the model, there are two basic routes through which a simple single-digit arithmetic problem such as 4 + 2 can be solved. In the first, direct route, which works only for overlearned addition and multiplication problems, the operands 4 and 2 are transcoded into a verbal representation of the problem ("four plus two"), which is then used to trigger completion of this word sequence using rote verbal memory ("four plus two, six"). This process is assumed to involve a left cortico-subcortical loop through the basal ganglia and thalamus. In the second, indirect semantic route, the operands are encoded into quantity representations held in the left and right inferior parietal areas. Semantically meaningful manipulations are then performed on these internal quantities, and the resulting quantity is then transmitted from the left inferior parietal cortex to the left-hemispheric perisylvian language network for naming. The model assumes that this indirect semantic route is used whenever rote verbal knowledge of the operation result is lacking, most typically for subtraction problems.

The two types of acalculic patients predicted by these two routes for calculation have now been identified (Dehaene & Cohen, in press). There are several published cases of acalculia following a left subcortical infarct (Whitaker, Habiger, & Ivers, 1985; Corbett, McCusker, Davidson, 1988; Hittmair-Delazer, Semenza, & Denes, 1994). In a recent case of a left lenticular infarct, Dehaene and Cohen (in press) showed that, as predicted by the model, only rote verbal arithmetic facts such as 3 × 3 were impaired (together with rote verbal knowledge of the alphabet, nursery rhymes, and prayers). Quantitative knowledge of numbers was fully preserved, as shown by the patient's intact number comparison, proximity judgment, and simple addition and subtraction abilities. Hittmair-Delazer et al. (1994) likewise observed a preservation of conceptual, quantitative, and algebraic manipulations of numbers in their severely acalculic patient with a left subcortical lesion.

Conversely, the model predicts that lesions of the left inferior parietal area, typically resulting in Gerstmann's syndrome, affect calculation because they destroy quantitative knowledge while preserving rote verbal abilities. Indeed, Dehaene and Cohen (in press) observed a Gerstmann-type patient who could still read aloud numbers and write them down to dictation, still knew rote addition and multiplication facts such as 2 + 2 and 3 × 3, and yet failed even in the simplest of quantitative tasks. He made 16% errors in larger–smaller comparison, 75% errors in simple subtractions (including gross errors such as 3 − 1 = 3 or 6 − 3 = 7), and 78% errors in number bisection (stating, for instance, that 3 falls in the middle of 4 and 8). The deficit was highly specific to the category of numbers, since the patient performed quite well in finding the middle of two letters, two days of the week, two months of the year, or two notes of the musical scale. We have now observed several such cases of severe number bisection deficits following a lesion of the inferior parietal area. These cases support the triple-code model's hypothesis that this area is critical for representing and manipulating number as quantities.

22-5. SOME OPEN ISSUES IN NUMBER PROCESSING

In spite of the progress made in understanding the architecture for number processing in the human brain, many gaps remain to be filled.

First, while we begin to understand the large-scale networks for number processing, further studies are needed to better understand the internal structure of representations, particularly the verbal representations used for accessing rote arithmetic facts.

Second, the issue of hemispheric specialization is still largely unsolved. Both hemispheres are clearly able to identify Arabic numerals and to represent the associated quantities. Yet, are the left and right representations equivalent, or are there differences between the magnitude representations available to the left and right hemispheres? Would the left hemisphere be categorical while the right would hold a more continuous, analogical representation of numbers? This important question remains to be addressed.

Finally, an issue that has emerged is the extent to which other types of mathematical knowledge involve yet other cerebral circuits than the ones we have described in this chapter. In addition to quantitative and rote verbal knowledge about numbers, most people also have encyclopedic knowledge (e.g., knowing that 1789 is the date of the French Revolution) and algebraic knowledge (e.g., knowing that $(a + b)^2 = a^2 + 2ab + b^2$). There are good indications that encyclopedic numerical knowledge can remain partially preserved in Gestmann's syndrome (Dehaene & Cohen, in press) and in patients with extensive left-hemispheric lesions (Cohen, Dehaene, & Verstichel, 1994). Hittmair-Delazer and her colleagues (1994, 1995) have even shown that conceptual algebraic knowledge was largely preserved in two severely acalculic patients. This observation suggests, against all intuition, that the neuronal circuits that hold algebraic knowledge must be largely independent from the networks involved in mental calculation. Their cerebral substrate, however, remains almost totally unknown.