Author's Response:

Is number sense a patchwork ?

Stanislas Dehaene

I would like to thank the commentators for raising excellent questions that illustrate the extent of what we do not yet know about the organization of the number system. The most critical comments seem to revolve around the multifaceted nature of the concept of number. Should we really postulate a single number sense, or is there a patchwork of representations and abilities, each underlying a specific set of tasks and experiments? If only for the sake of economy, I would like to defend the idea that a single, analog representation of quantities underlies the core of our numerical abilities and intuitions. In the course of development and education, however, this central representation becomes connected to other cognitive systems, including word comprehension and production and object tracking devices. What is at stake is to what extent the initial quantity system is altered through these interactions; and to what extent the adult concept of number is based solely on the quantity system as opposed to an integration of multiple senses of numbers.

Comparison: Is the number line solely a cultural construction ?

Giaquinto argues the number line is a "culturally supplied representation", a visual tool learned from school that we use to convene a concrete image of numbers, and that must therefore be distinguished from the biologically inherited quantity representation that underlies number sense. He further argues that the distance and magnitude effects that are seen in the number comparison task "could well be accounted for in terms of the operations of 'scanning' and 'zooming in' postulated by Kosslyn in his theory of the visual imagery system". If it was literally true that number comparison relies on visual imagery, would deprive the concept of an analog quantity representation of some of its strongest empirical support. I believe that there is much to say against it, however. Preschoolers exhibit an even stronger distance effect than adults when comparing Arabic numerals (Temple & Posner, 1998). Monkeys also exhibit magnitude and distance effects of a highly comparable form when comparing Arabic symbols and sets of physical objects (Brannon & Terrace, 1998; Brannon & Terrace, 2000; Washburn & Rumbaugh, 1991). It seems exceedingly unlikely that preschoolers and monkeys have been exposed to the cultural concept of 'number line' and are able to bring up a visual mental image of a ruler or a similar device. Furthermore, brain-imaging studies provide no support for the activation of visual imagery processes in the occipital or inferior temporal regions during number comparison (e.g. Chochon, Cohen, van de Moortele, & Dehaene, 1999), nor would there be any time for the conjuration of a visual image in the 400-600 ms that it typically takes to compare two digits. Finally, there is evidence for an abnormal or absent distance effect in some patients with dyscalculia (Butterworth, 1999), although there is no reason to think that these patients have impaired visual imagery process.

The evidence from the number comparison task suggests that the distance and magnitude effects are integral properties of the number representation, rather than artifacts of the comparison task and of a putative accompanying imagery component. It is possible that something like zooming and scanning operations underlie quantity manipulations; but they must be occurring on this abstract quantity representation, not a generic visual medium. Obviously, the number line is only a metaphor for the organization of this representation. No one thinks that number-coding neurons are lining up neatly next to one another in parietal cortex, in increasing order! Like **Pesenti and Seron,** I would perfectly happy if the quantity representation was found to be implemented by distributions of activations that overlapped more for close numbers than for far numbers, and all the more so that the numbers are large (this is exactly what was proposed in Dehaene & Changeux, 1993).

The number line metaphor is appropriate, however, because the spatial concepts of distance and proximity readily apply to the metric structure of semantic similarities between numbers; because multidimensional scaling shows that this similarity matrix is best captured by a one-dimensional line (Shepard, Kilpatrick, & Cunningham, 1975); because naive subjects as well as professional mathematicians spontaneously use spatial words spontaneously when speaking about numbers; because a few subjects even claim to experience 'number forms', spatial images of number lines that have no obvious cultural origins (Galton, 1880; Seron, Pesenti, Noël, Deloche, & Cornet, 1992); and because, as discussed by **Giaquinto** as well as by **Pesenti and Seron**, the SNARC effect reveals that Arabic numeral automatically evoke a spatial left-to-right bias congruent with their quantity (Dehaene, Bossini, & Giraux, 1993). The fact that the direction of this bias is culturally dependent, or that it can be reversed when imagining a clockface (**Giaquinto**) is irrelevant here. What must be explained is why an association between number and spatial concepts is found at all, and even in neutral situations with no spatial imagery component (e.g. when having to judge how the corresponding number words sound; Fias, Brysbaert, Geypens, & d'Ydewalle, 1996). The metaphorical mapping between numbers and space is a cross-cultural universal that underlies basic proto-mathematical concepts such as scaling and measure as well as purely mathematical constructions such as Cartesian coordinates. There is no such deep connection to spatial concepts that I know of in the case of other domains such as color, or shape. In spite of **Giaquinto**'s disagreement, I maintain my postulate that a natural mapping between numbers and space arises because nearby and presumably similar neural structures in the parietal lobe are involved in the representation of both domains.

Subitizing: Is there a separate system for very small numbers ?

Another possible separation, endorsed in three comments (**Carey, Giaquinto, Pesenti and Seron**), attempts to distinguish the analog quantity representation from a system of object files dedicated to the very small numbers one, two, three, and perhaps four. **Carey** argues that this object file system, not the quantity system, lies at the foundation of the ontogeny of the number system in young children. Here again, I would caution against dividing up the number system too early, before the evidence compels us to do so. There is certainly quite a bit of evidence against the notion of a qualitatively distinct system for small numbers. First, the adult subitizing evidence, as obtained in a task of naming visually presented numbers of dots, does not imply a distinct system for small numbers. Rather, it can be explained by a single continuous system of number estimation whose precision decreases for large numbers following Weber's law. According to this explanation, the subitizing range is simply the range over which subjects feel confident enough that they can name the number accurately without counting; and the subitizing limit is the first point at which subjects start to count. Congruent with this explanation, when the task does not require precise naming, but merely the non-verbal discrimination of sets of dots, no discontinuities emerge (van Oeffelen & Vos, 1982).

Second, Brannon and Terrace's research with monkeys (Brannon & Terrace, 1998; Brannon & Terrace, 2000), reported in my *précis*, not only shows again an absence of discontinuity, but in fact a complete generalization of training from numerosities in the range 1-4 to novel numerosities in the range 5-9. Once monkeys were trained to point to numerosities in the order 1,2,3,4, they spontaneously generalized this ordering to larger numerosities such as 7 versus 9 (again with distance and magnitude effects). Such behavior would be impossible to explain if the representation used for ordering small numbers of objects was dissociated from the one used for larger numbers. In passing, this experiment answers most of the worries raised by **Pesenti and Seron**. Those commentators correctly point out that most experiments on number in animals involve laboratory training rather than testing of natural competence in the field, and that it is hard to rule out all non-numerical confounds. In anticipation of the second query, Brannon and Terrace’s experiment includes novel and extreme variations in object disposition and identity, thus convincingly demonstrating that monkeys’ behavior is based on number ; and, although it falls under the first criticism (laboratory training), the observation of spontaneous generalization to a different range of numerosities indicates that the animals spontaneously brought to bear on the task much more than what the training phase could possibly have inculcated them. The experiment reveals, rather than teaches, a representation of numbers and their order that does not stop at number 4.

Superficially, one could take the remarkable evidence presented in **Hurford**’s commentary as indicative of a distinct system for small numbers. Over the years, Hurford gathered thorough information about the linguistic properties of the number words, and discovered that small numbers were often given special treatment within the language system (think, for instance, of the markers of singular, dual, and even trial as opposed to plural in some languages; or the exceptions to the rules for forming ordinals, e.g. the words ‘premier’ and ‘second’ in French). In my book, I cited Hurford’s (1987) book, which suggested that exceptions were found only for the numbers 1, 2 and 3. The newer and more complete evidence reported in Hurford’s commentary, however, also lists occasional exceptions for numbers 4 and 5. Most importantly, as shown in his table 1, there is no clear discontinuity at any point. What is observed is a progressive, exponential decrease in the number of exceptional word properties, quite similar to the continuous exponential found for the decrease in the frequency of number words, which is also universal across languages

(Dehaene & Mehler, 1992). In his commentary, Hurford seems to agree that an analog representation with a continuously decreasing precision suffices to explain his data.

The heart of the dispute about object files arises from the difficulty of interpreting infant and animal data. Clearly, something like an object file system is needed to keep track of objects in simple addition and subtraction experiments *à la* Wynn (1992). Clearly too, this system may contribute noise or even go against the ability to discriminate number (as in Xu and Carey's (1996) experiment, where infant's tracking system fails to use large differences in object identity to infer the presence of two distinct objects). What is not clear to me is whether this system alone serves as the ontogenetic foundation of numerical development, as proposed by **Carey**. There is already much evidence that the object tracking is not sufficient. For instance, it cannot account for infant's discimination of 2 vs. 3 auditory events (Bijeljac-Babic, Bertoncini, & Mehler, 1991), 2 vs. 3 puppet jumps (Wynn, 1996), or 8 vs. 16 visual objects (Xu & Spelke, 2000). Thus, Carey agrees that we need to postulate a distinct genuinely numerical system, presumably analog and obeying Weber's law. Strangely enough, however, this system would play no role whatsoever in conceptual development. This seems unparsimonious, especially given that we have seen that a system with continuously decreasing precision could often make it appear, behaviorally, as if there was a discontinuity around 3 or 4. I would argue that some of the evidence that **Carey** presents, such as Karen Wynn's (1990) study of the sudden grasp of the number counting sequence starting at number four in 3 ½ year olds, can be accounted for by a continuous system in exactly the same way as the adult subitizing data can. Given the imprecision of the analog system, 4 seems to be the first point at which a number cannot be distinguished from its successor in an essentially error-free manner. Hence, it is the first point at which children experience a direct conflict between count words and their internal analog representation, forcing them to re-think this mapping and come up with a generic solution for all the number words that they know.

While other infant and animal data do pose some problems, it is unclear to me whether we have sufficient evidence, and especially sufficient knowledge of the limits of experimental paradigms, to assess the real significance of the observed discontinuities and apparent violations of Weber's law. For instance, could the limit of 3 items in Hauser et al.'s monkey foraging experiments (Hauser, Carey, & Hauser, 2000), and similar evidence in infants (Feigenson and Carey, cited in Carey's commentary) simply reflect a lack of motivation for looking for more than 3 food items in the conditions of the experiments ? Or from the fact that number and duration of stimulus presentation are confounded, resulting in higher working memory requirements on trials with larger numbers ? **Carey** builds a strong case out of the fact that Weber's law is violated in infants because they discriminate 2 from 3, but not larger numbers in the same ratio such as 4 from 6 (Starkey & Cooper, 1980) or 8 from 12 (Xu & Spelke, 2000). However, do we know enough about Weber's law and its development to interpret this pattern as reflecting the existence of two systems, rather than one system that would show a non-linear deviation from Weber's law early in its maturation ? There are some inconsistencies in the ratios obtained by different experimenters with different methods (e.g. success in discriminating 8 from 16 (Xu & Spelke, 2000), but failure in computing 5+5=10 or 5 (Chiang & Wynn, in press), also a 2:1 ratio). While waiting for a systematic study of Weber fractions for different number sizes as a function of age, the scarcity of available infant data suggests that great caution is needed before drawing strong conclusions.

Language and symbols: What are their roles in adult calculation ?

Regardless of how the systems of number words, Arabic symbols, and quantities develop in children, there is no doubt that most adults have at their disposal all three systems and a quick and accurate interconnection between them. It seems likely that the development of an interconnection with exact symbol systems significantly alters the quantity system. There is little direct evidence on this topic, but the fact that the Weber fraction observed in a purely non-verbal test of number discrimination is about twice smaller in human adults than in rats (Mechner, 1958; Whalen, Gallistel, & Gelman, 1999) hints that the precision of the analog representation may be increased as a result of connecting it with an exact system of symbols. In *The Number Sense*, I attributed the unequalled achievements of *homo Sapiens* in mathematics to the specifically human competence for creating symbol systems (whether in spoken language or in writing) and connecting them to evolutionarily ancient nonverbal representations such as the quantity system.

My triple-code model of number processing postulates that calculation relies on an interplay between the language, Arabic, and quantity systems. Some arithmetic facts, such as overlearned multiplications, are simply stored in rote verbal memory (e.g. three times nine is twenty-seven); others, such as simple subtractions, are generally not remembered verbally and require a semantic manipulation of quantities (e.g. 3-1=2). Yet others require a collaboration between both systems (e.g. when we compute 8x9 as 8x10 – 8). Finally, multi-digit calculation is assumed to recruit the Arabic system for laying down the operations mentally as on paper. **Giaquinto** disputes my postulate that linguistic representations play an important role in calculation using two lines of evidence : the case of a languageless autistic young man, Michael, who could factorize numbers and compute primes (Hermelin & O'Connor, 1990), and the case of a global aphasic who could still calculate (Rossor, Warrington, & Cipolotti, 1995). Both cases are quite extreme, however, and therefore unable to challenge my hypothesis that language representations play a central role in *normal* calculation in most cultures. Michael, the autistic calculator, obviously had learned to bypass his lack of spoken language by relying on the written symbol system of Arabic digits; hence, he certainly could not be said to calculate without symbols. The aphasic patient also relied on Arabic numerals, though interestingly, exactly as predicted by the triple-code model, his retrieval of stored multiplication facts was impaired and he had to rely on back-up semantic strategies.

Training with the abacus in Asian countries clearly indicates that there are alternatives to language-based arithmetic. Even in those countries, however, rote verbal storing of multiplication facts ('*kuku*' in Japan) also plays a central role. My claim is simply that, in trying to solve the difficult task of performing arithmetic computations, the brain makes use of whatever representations are most helpful. The finding of dissociations between operations, and particularly of severe multiplication deficits and preserved subtraction in several patients with language-related deficits (e.g. Cohen & Dehaene, 2000; Dehaene & Cohen, 1997), supports the idea that rote verbal memory is a useful such resource for most of us.

Genius: How do we go from basic arithmetic to higher mathematics ?

In *The Number Sense*, I argued that the remarkable abilities exhibited by prodigious calculators and professional mathematicians do not necessarily force us to grant them a different brain architecture. I speculated that training, fueled by a passion for the subject topic, could enhance pre-existing mental representations and their interconnections until they became so fluid as to seem exceptional and of a different nature. **Giaquinto** expresses skepticism at this hypothesis. Part of our disagreement, however, stems from a misunderstanding. I never meant that someone could become a top-level mathematician by practicing *calculation*; this would be absurd, as most of mathematics is not based on calculation and many mathematicians are normal or even poor calculators. As in the rest of the book, I was using calculation as an example where evidence is available to show that, in this domain at least, training and passion can turn any normal person into an outstanding ‘genius’. That seems relatively uncontroversial. Training experiments can enhance normal students’s numerical memory and calculation abilities to ‘prodigious’ levels (Chase & Ericsson, 1981; Staszewski, 1988). Binet’s (1981)studies of the *Bon Marché* accountants indicate that a few years of intense practice can turn normal people into fast calculators on a par with the well-known prodigy Inaudi. Pesenti et al. (1999) shows that normal effects of number size and distance are present in a calculating prodigy, only reduced in size. Finally, the algorithms that are used by autistic prodigies are not nearly as mysterious as **Giaquinto** thinks. For instance, Michael, the autistic calculator, seemed to decide whether a number was prime by applying the well-known rules of divisibility by 2, 3, 5, 9, and 11, because he made false alarms on numbers that were not primes, but were composed of large factors (e.g. 13 x 17). I would also warn against the dangers of hagiography in this domain, as many of the most extreme feats attributed to prodigies, and precisely those that would challenge known cognitive mechanisms (e.g. instant factoring of 6-digit numbers, or exact perception of numerosities in the hundreds without counting) are not reliably documented.

By analogy with the number domain, then, can we imagine that the great mathematicians have developed their expressive powers through extensive training of pre-existing representations that we all share, such as spatial maps, visual imagery, or symbolic processing? I still think that this is plausible. What we know from neuroscience is that experience in early childhood (similar to Mozart’s?) can have much more dramatical and longer-lasting effects on neural development that later training (e.g. Knudsen & Knudsen, 1990); and that stimulation of neuromodulator circuits involved in attention and arousal concomitant with experience can potentiate brain plasticity and cause a massive invasion of cortical maps by a representation of the attended stimuli (e.g. Kilgard & Merzenich, 1998). When Giaquinto argues that there must be « an independent dimension of variability, unknown to us at present, which explains the differences » between great mathematicians and the rest of us, my hunch is that early experience-dependent brain plasticity is a good place to look. Not, of course, that I endorse the extreme empiricist attitude adopted by many neuroscientists involved in plasticity research (e.g. Quartz & Sejnowski, 1997). Experience does not direct neural growth or leave imprints of novel representations on a cortical '*tabula rasa*'. Rather, existing representations can be modulated in their extent, sharpness, and connectivity both within a given area and between areas, thus altering the speed and ease with which the same cognitive operation can be performed and, in the end, causing seemingly incommensurate differences in cognitive abilities.

This naturally leads to **Hurford**'s question concerning the anatomical variability of the neural substrates for arithmetic and mathematics. Given that mathematics is a cultural activity that depends heavily on teaching and education, some amount of variability in cerebral representation is to be expected. I do not, however, share Hurford's pessimistic idea that, as we gain more insights into brain function, the "neat correspondences betwen brain areas" and representational codes will be found much more variable than I propose. On the contrary, with the advent of fMRI and its ability to measure activity in single subjects with millimeter resolution, we are beginning to realize that the variability is *smaller* that we initially expected. Even within the notoriously imprecise coordinate frame of the Talairach system, which does not compensate for inter-individual variations in sulcal and gyral anatomy, the standard deviation of the coordinates across several individuals can be as small as a few millimeters in the fusiform face area (Kanwisher, McDermott, & Chun, 1997) or the visual word form area (Cohen et al., 1999). In the case of language, which is used by Hurford as an example, the apparent lack of reproducibility of the association between lesion sites and syndromes could be due to our current misunderstanding of the relevant brain specialization.

Once a specialized system is isolated, however, such as the anterior insula circuit for speech articulation, its localization can be remarkably reproducible (Dronkers, 1996). What seems to more variable, and more affected by cultural variations, is the extent of the activations and the relative strengths of their various components depending on the strategies used (Paulesu et al., 2000). The core circuits, however, can be reproducibly found in all individuals.

Metaphysics: Do numbers and other mathematical objects ‘exist’ in the physical world?

I would like to end this response by answering some of **Giaquinto**’s remarks on the metaphysics and epistemology of arithmetic. Whether we distinguish three or four classes of philosophical attitudes on the nature of numbers, no such scheme can ever do full justice to the subtleties of the philosophical positions held by specific individuals. I therefore thank Giaquinto for his accurate clarification of Hilbert’s views about the nature of mathematical objects, to which I could not possibly do justice in my book. What I wanted to discuss, however, is to what extent numerical cognition research bears on the philosophical issues of the ‘reality’ and ‘existence’ of mathematical objects. Numerical cognition research suggests a plausible route for how the concept of number is constructed by the human brain and mind. This construction is not arbitrary, however, and here Giaquinto distorts my views by suggesting that I believe that “our number sense detects nothing external to its own output”. My proposal is that the brain evolved a number system to capture a significant regularity of the outside world, the fact that at our scale, the world is largely composed of solid physical objects that move and can be grouped according to the laws of arithmetic.

I believe that it is indispensable, for reasons of epistemological coherence, to maintain a sharp distinction between models and reality, and therefore between mathematical objects and the regularities of the physical world that they capture. Failing to do so leads to the quick sands of Platonism, with its cohort of paradoxes. **Giaquinto** seems to think that number 2 ("the number of planets with orbits smaller than Earth's") exists in the physical universe. Perhaps one could grant this, but quickly one would have to ask whether other derived mathematical objects also 'exist' out there. What about the square root of 2? The square root of *minus* 2? Or Cantor's aleph-2? Where does reality stop, where does cultural construction start?

The case of number clearly lies at an extreme, and that is perhaps why my proposal seems counter-intuitive. The presence of detachable objects that form sets is such a striking physical regularity (to us at least) that it is hard not to attribute a degree of 'reality' to their cardinal. And indeed, this cardinal does not depend solely on the human mind since many animal species recognize the same numbers as we do. In this sense, the number system is much more stable than the color perception system, which varies from species to species. It might seem innocent, then, to grant, with **Giaquinto**, that "cardinal numbers are objective and independent properties of sets". I can only reiterate here, however, the cautionary statements I wrote near the end of TNS (pp. 251-252):

> "Platonism hits upon an undeniable element of truth when it stresses that physical reality is organized according to structures that predate the human mind. However, I would not say that this organization is mathematical in nature. Rather, it is the human brain that translates it into mathematics. The structure of a salt crystal is such that we cannot fail to perceive it as having six facets. Its structure undeniably existed way before humans began to roam the earth. Yet only human brains seem able to attend selectively to the set of facets, perceive its numerosity as 6, and relate that number to others in a coherent theory of arithmetic. Numbers, like other mathematical objects, are mental constructions

whose roots are to be found in the adaptation of the human brain to the regularities of the universe".

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