



Breaking down number syntax: Spared comprehension of multi-digit numbers in a patient with impaired digit-to-word conversion

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ABSTRACT

Can the meaning of two-digit Arabic numbers be accessed independently of their verbal-phonological representations? To answer this question we explored the number processing of ZN, an aphasic patient with a syntactic deficit in digit-to-verbal transcoding, who could hardly read aloud two-digit numbers, but could read them as single digits (“four, two”). Neuropsychological examination showed that ZN’s deficit was neither in the digit input nor in the phonological output processes, as he could copy and repeat two-digit numbers. His deficit thus lied in a central process that converts digits to abstract number words and sends this information to phonological retrieval processes. Crucially, in spite of this deficit in number transcoding, ZN’s two-digit comprehension was spared in several ways: (1) he could calculate two-digit additions; (2) he showed good performance in a two-digit comparison task, and a continuous distance effect; and (3) his performance in a task of mapping numbers to positions on an unmarked number line showed a logarithmic (nonlinear) factor, indicating that he represented two-digit Arabic numbers as holistic two-digit quantities. Thus, at least these aspects of number comprehension can be performed without converting the two-digit number from digits to verbal representation.

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1. Introduction

Benjamin Lee Whorf suggested that language lies at the core of human thought and shapes our concepts (Whorf, 1940). In the domain of arithmetic, this view has been explicitly refuted by showing that a broad array of numerical abilities are

spontaneously present even without verbal representation of numbers. This has been demonstrated in animals, preverbal infants, and adults from language communities with a reduced lexicon of number words (Brannon & Terrace, 2000; Dehaene, Izard, Spelke, & Pica, 2008; Dehaene, Molko, Cohen, & Wilson, 2004; Feigenson, Dehaene, & Spelke, 2004; Hauser, Carey, & Hauser, 2000; Nieder & Dehaene, 2009; Viswanathan &

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Nieder, 2013). Yet a narrower hypothesis may still be tenable, according to which some higher mathematical abilities are tightly coupled with language: a specifically human recursive computation mechanism may underlie syntactic processes, not only in language, but in other cognitive processes, including the way we represent multi-digit numbers and mathematical expressions (Hauser, Chomsky, & Fitch, 2002; Houdé & Tzourio-Mazoyer, 2003). Even this view of “global syntax”, however, is challenged by certain findings: brain areas and functional processes that characterize language syntax are dissociable from those that support many combinatorial mathematical processes, including the processing of algebraic operations (Monti, Parsons, & Osherson, 2012) and mathematical expressions (Maruyama, Pallier, Jobert, Sigman, & Dehaene, 2012), multi-digit number naming (Brysbaert, Fias, & Noël, 1998), transcoding, and multi-digit calculation (Varley, Klessinger, Romanowski, & Siegal, 2005).

The present study aims to further probe this issue by analyzing dissociations between different syntactic processes within the domain of number cognition in an aphasic patient with impaired conversion of Arabic numbers to words. Specifically, we asked whether the meaning of two-digit Arabic numbers can be accessed independently of their verbal representations when the syntactic mechanisms converting numbers to words are impaired. Our goal is to examine in detail the locus and nature of the patient's impairment in transcoding, and to evaluate his number meaning abilities and syntactic processes of number comprehension using various tasks.

Numbers have three distinct representations: they can be coded in digits as Arabic numerals (68), as number words (sixty-eight), or as quantities, the dominant “meaning” of the number. These different cognitive representations are dissociable (Gordon, 2004; Lemer, Dehaene, Spelke, & Cohen, 2003), can be selectively impaired (Cohen & Dehaene, 2000), and are implemented in different brain areas (Dehaene, Piazza, Pinel, & Cohen, 2003). However, these representations are tightly related: symbolic representations of numbers (words, digits) are associated with the corresponding quantities, which can be represented spatially along a left-to-right mental number line (Dehaene, Bossini, & Giraux, 1993; Loetscher, Bockisch, Nicholls, & Brugger, 2010; Moyer & Landauer, 1967; Ruiz Fernández, Rahona, Hervás, Vázquez, & Ulrich, 2011; Shaki, Fischer, & Petrusic, 2009).

Multi-digit Arabic numerals enter into several types of internal conversion processes. Converting multi-digit Arabic numbers to number words is a syntactic process that requires encoding the relative positions of the digits according to the base-10 system, converting each digit to a word according to its position, and combining the words, sometimes with the addition of coordination markers (“and”). This syntactic sub-process can be selectively impaired (Cipolotti, 1995; Noël & Seron, 1993). But multi-digit Arabic numbers can also be quickly converted into the corresponding quantity (Dehaene, Dupoux, & Mehler, 1990; Dotan & Dehaene, 2013; Reynvoet & Brysbaert, 1999). Computing the quantity associated with an Arabic multi-digit number requires encoding the relative positions of the digits and combining their quantities according to the base-10 principles. Thus, syntactic operations are required when converting multi-digit Arabic numerals either to verbal number words or to quantities. Is a single syntactic

process involved in both conversion processes? The present study addresses one aspect of this question: we asked whether the conversion of a two-digit Arabic number into a quantity can be spared when the syntactic operation involved in forming a verbal representation of the number is impaired.

The triple-code model of number processing (Dehaene, 1992; Dehaene & Cohen, 1995) predicts that there is a direct conversion route from Arabic inputs to the quantity representation, independent of the Arabic-to-verbal route. However, the verbal representation of numbers is also thought to play a crucial role even in tasks that do not necessarily involve overt comprehension and production of verbal numbers, e.g., memorization of arithmetic facts (Cohen & Dehaene, 2000; Dehaene, 1992; Dehaene & Cohen, 1997). The relation between verbal representations and quantity is sometimes surprisingly complex, to the extent that quantity encoding may be affected by the language in which a verbal number is presented, even in the same person: Dehaene et al. (2008) investigated individuals from an Amazonian culture with little or no formal mathematical education, and found that their quantity processing showed a more linear pattern when numbers were presented in a Western second language (Portuguese) than in their native tongue, Mundurucu, where numerals yielded a more logarithmic pattern.

To separate these two possibilities, and probe whether Arabic-to-quantity conversion makes use of a syntactic process that is also needed for Arabic-to-verbal conversion, we examined the various number abilities of ZN, an aphasic patient who has a selective deficit in verbal number production. This deficit prevents him from converting multi-digit numbers into verbal-phonological forms, and renders him almost completely unable to say them aloud. We tested whether, in spite of this deficit, he can encode the holistic quantity of two-digit Arabic numbers.

Another question addressed in this study is whether multi-digit addition depends upon verbal-phonological forms of number words. Rote knowledge of arithmetic facts relies on the verbal representation of numbers (Cohen & Dehaene, 2000; Dehaene, 1992; Dehaene & Cohen, 1997; Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999). Multi-digit addition too can be affected by verbal factors such as the grammatical structure of number words (Colomé, Laka, & Sebastián-Gallés, 2010), though not always (Brysbaert et al., 1998). Phonological representations may be involved in multi-digit addition, but are probably not necessary for it (Klessinger, Szczerbinski, & Varley, 2012). The present case study of ZN extends Klessinger et al.'s conclusions by examining a dissociation between multi-digit addition and the verbal representation of numbers: we tested whether ZN could solve addition problems that involve two-digit numbers in spite of his severe impairment in the conversion of Arabic numbers to verbal representation.

2. Case description

2.1. Overview

ZN, a 73-years-old man, used to work as an engineer, a job that involved a lot of number processing. When he was 72

he had a subacute infarct in the left corona radiata, following which he lost much of his speech, and was diagnosed with aphasia, severe apraxia of speech, impaired comprehension, dyslexia, dysgraphia, and agrammatism. He started language rehabilitation that focused on articulation, lexical retrieval, and grammatical processing. By the time we met him, eleven months after his stroke, he still had severe difficulties in comprehension and production of speech. Until that time he was neither diagnosed nor treated for number processing.

ZN is right-handed and wears reading glasses. His mother tongue is Hebrew, and all tests were conducted in this language. He is an engineer with B.Sc. degree. We tested him in a series of 45-min sessions that took place in a quiet room in his home. The sequence of sessions lasted several months, but crucial tasks (hereby described) of number reading and comprehension were administered in intertwined sessions within a short period of less than 3 months.

2.2. Language assessment

ZN's lexical retrieval was impaired. When asked to name 100 objects in a picture naming task (SHEMESH, Biran & Friedmann, 2004), he made phonological errors and neologisms in 45/100 items. These could be explained by his apraxia of speech, but he also failed to make any verbal response to 40 items, and made 13 semantic errors – a finding that indicates that on top of his apraxia, he also had a deficit in an earlier stage of lexical retrieval (Friedmann, Biran, & Dotan, 2013).

His working memory was assessed in a digit span task (Friedmann & Gvion, 2002; Gvion & Friedmann, 2012) in which he answered by pointing to the digits 0–9 on paper. His digit span was 3, significantly lower than an age-matched control group ($z = -3.06$, $p = .001$; control data from Gvion & Friedmann, 2012).

ZN had severe morpho-syntactic difficulties. In a picture-to-sentence matching task (BAFLA, Friedmann, 1998), his performance was at chance level not only in object and subject relative clauses (18/40 errors), but even in simple subject-verb-object sentences (14/30 errors). In a sentence completion task, which required him to inflect verbs for tense and agreement (BAFLA, Friedmann, 1998), he had inflection errors in 10/24 items and failed to respond to 5 items. The morphological difficulty was present not only in verbs but also in nouns: in a picture-naming task that required ZN to inflect morphologically complex nouns, he made morphological errors (substitution or omission of the morphological affix) in 7/20 items and failed to respond to 2 items.

His reading aloud was impaired too. In reading 49 words from the TILTAN dyslexia screening test (TILTAN, Friedmann & Gvion, 2003), he made 37 errors and failed to respond to 6 additional words. His errors were phonological paraphasias (phonemic and formal, in 20 items), neologisms (10 items), morphological errors (8 items), and sublexical reading (surface-dyslexia-like errors, 4 items). His word writing was severely impaired too (8/11 errors).

3. Assessment of symbolic number processing

3.1. Input and output of Arabic numbers

ZN's ability to read and write numbers as digits was assessed using two tasks: number dictation and delayed copying of numbers.

Number dictation: The experimenter said aloud the 40 numbers between 1 and 40 in random order, one at a time, and ZN wrote them as Arabic numerals. He performed this task without any error (40/40 correct).

Delayed copying: ZN saw twenty 2-digit numbers on the computer screen, one at a time. Each number was presented for one second, and after it disappeared, ZN was requested to write it down on paper. To eliminate possible verbal rehearsal, ZN was required to say aloud the first two Hebrew alphabet letters (*alef*, *bet*) after the target number disappeared from the screen and before he wrote it down. In this task too, his performance was flawless (20/20 correct).

These two tasks demonstrate ZN's ability to write two-digit numbers in Arabic notation, which stands in contrast to his complete inability to write words (Fisher's $p < .001$ in number dictation vs word writing). The delayed copying task further shows that his Arabic number input is intact: he correctly encoded the identity and the relative positions of the digits in the two-digit number, and could memorize the number until writing it down. The dictation task shows his preserved ability of converting number words to digits.

3.2. Production of verbal number words

Next, we assessed ZN's ability to produce number words orally. In the following tasks we classified responses as correct whenever the target number was produced, even if it was not articulated completely accurately (e.g., 35 → “thirty five”). This is because our focus in this section was not on examining ZN's articulation, which is known to be impaired by his apraxia of speech, but on examining earlier processing stages involved in number processing. Thus, phonological errors were not included in the overall error rates.¹

Reading aloud 2-digit numbers: ZN saw a list of 33 two-digit numbers and was asked to read them aloud. The numbers were administered as three different lists (of eight, five, and 20 numbers) in separate sessions. Five of the numbers were teens and the rest were larger than 20. None of the numbers included the digit zero. The first two lists were printed on paper in Arial 22 font. The third list (of 20 numbers) was read from the computer screen, and the numbers were presented for 1 s.

In marked contrast to his good number writing, ZN's reading aloud was very poor and he produced correctly only 7

¹ In reading aloud 2-digit numbers (the task hereby described), ZN made 15 phonological errors of the 25 numbers that were encoded for phonological errors (we had technical problems with the audio recording of the remaining 8 items, so we do not know how many phonological errors they included). In reading 2-digit numbers as single digits (the task described next), he made phonological errors in 20/71 digits. In number repetition he made phonological errors in 30/40 items.

of the 33 numbers. The most striking error pattern was that he read most of the numbers as two separate digits (e.g., 15 → *one five*, or 47 → *four seven*). This occurred for 23 of the 33 items (70%). He was able to produce the decade name for only 9 of the 28 numbers larger than 21, and could not produce the teen form for any of the five teen numbers that were presented. On top of that, he had other errors too: he omitted a digit in two numbers, made lexical within-class errors in 14 numbers (e.g., 63 → *seventy three*), and class errors in 2 numbers (e.g., 14 → *forty*; in Hebrew, some class errors are not phonologically similar to the target, and this was the case with these two errors).

Reading 2-digit numbers as single digits: ZN was presented with a list of the 40 numbers from 1 to 40, appearing in random order, and was asked to read them aloud as single digits (e.g., 54 → *five*, *four*) – i.e., a total of 71 digit names. The numbers were printed on paper in Arial 22 font. ZN performed well in this task: he made only one lexical error, and another error that could be interpreted either as lexical or as phonological, with a total of 69/71 correct digit names.

Thus, although ZN was able to identify and name the digits in two-digit numbers, he was unable to say aloud two-digit numbers when he was asked to produce the number name with a valid decade + unit syntactic structure (5% vs 79% errors, $\chi^2 = 41.6$, $p < .001$). This pattern persists even when comparing only the 15 two-digit numbers that appeared in both tasks (one error in the digit reading task, but 13 errors when reading the numbers with a valid syntactic structure, Fisher's $p < .001$). Namely, when he read a number as a two-digit number, he failed, but when he read the exact same number digit by digit, without the need to process its syntax, he succeeded. In the next section, we explore the origin of this multi-digit number production deficit.

3.3. The origin of ZN's difficulty in verbal number production

The results show that ZN has a deficit in transcoding Arabic numbers to number words that selectively affects his ability to verbally produce two-digit (and multi-digit) numbers. This deficit is not in the input stages, as demonstrated by ZN's good performance in the delayed copying task, by his ability to read two-digit numbers when required to say only the digit names, and, as we will show below (Section 4.1), by his spared ability to perform two-digit additions. One crucial result in this matter is his good production of single digits (e.g., “four, three” for 43). Is it the case that ZN's deficit is in the production stage, and he cannot retrieve teen and decade number words? If this were the case, we would expect him to fail also in other tasks that require multi-digit number production, such as number repetition. We therefore tested his multi-digit number repetition.

Number repetition: The experimenter read aloud the numbers between 1 and 40 in random order and ZN was asked to repeat the number. ZN's ability to repeat numbers correctly (or only with phonological errors) was good, and clearly superior to his performance in the number reading task: he correctly repeated 37 of the 40 numbers, and made one lexical error, one error that may be lexical or syntactic, and one lexical or phonological error. Importantly, in the repetition task

ZN never tried to produce the digit names instead of the number name, like he so often did in the number reading task.

Any task that requires number production involves retrieval of the phonological forms of the number words from a phonological storage in the phonological processing stages (Dotan & Friedmann, 2007, 2010, *in press*; McCloskey, Sokol, & Goodman, 1986). ZN's good repetition shows that he was able to retrieve the phonological forms of the number words and to articulate them (even if with phonological errors). Thus, his phonological retrieval and articulation stages are not the source of his difficulty to produce two-digit number words. The deficit that caused this difficulty must be in a pre-phonological processing stage, between the input of the written numbers and their production. This conclusion, and the evidence supporting it, is visually summarized in Fig. 1.

Number-like nonword repetition: Finally, we ruled out a possibility that ZN's good performance in the number repetition task was due to phoneme-by-phoneme repetition, and therefore may not indicate that he was able to retrieve the phonological representations of numbers. To assess this possibility, we compared his performance in number repetition with a nonword repetition task, which is undoubtedly performed phoneme-by-phoneme, via the sublexical repetition route. For each number word, a corresponding nonword was created with matched length, syllable structure, stress position, morphological structure, CV structure, and consonant clusters. For example, for the number 34, /shloshim ve-arba/, the matched nonword was /frifol ve-umdi/. The order of the stimuli was also the same in both tasks. This created a list of 40 number-like nonwords.

The comparison between the repetition of numbers and number-like nonwords revealed very different patterns. ZN made significantly more consonant substitutions in nonword repetition than in number repetition (13% vs 4% out of 180 consonants, $\chi^2 = 9.3$, one-tailed $p = .001$). This indicates that the two tasks were performed via different processing pathways: the number repetition task involved a lexical repetition route, which induced a smaller amount of phonological errors than the sublexical repetition route. The findings therefore refute the possibility that ZN repeated numbers sub-lexically, and support the conclusion that he can retrieve number words, including teens and decades, when the access to the phonological number words does not involve reading of a multi-digit Arabic number, and hence, does not require digit-to-verbal transcoding.

Another finding that refutes a strictly sublexical number repetition hypothesis concerns the way ZN combined the decade and unit names. In Hebrew, the decade and unit number words are combined by the function word “and” (i.e., we say “thirty and two” rather than “thirty two”). The Hebrew word “and” is generally pronounced as /ve/, but in some phonological contexts it is considered normatively “more correct” or higher register to pronounce it as /u/. In the repetition task, the experimenter used the /u/ pronunciation in three occasions, and in all cases ZN repeated the number using the /ve/ pronunciation, thereby showing that he did not process the function word merely as a sequence of phonemes, but treated it as a lexical/syntactic element. Together with the different patterns of phonological errors in number and nonword repetition, these results indicate that ZN did not use

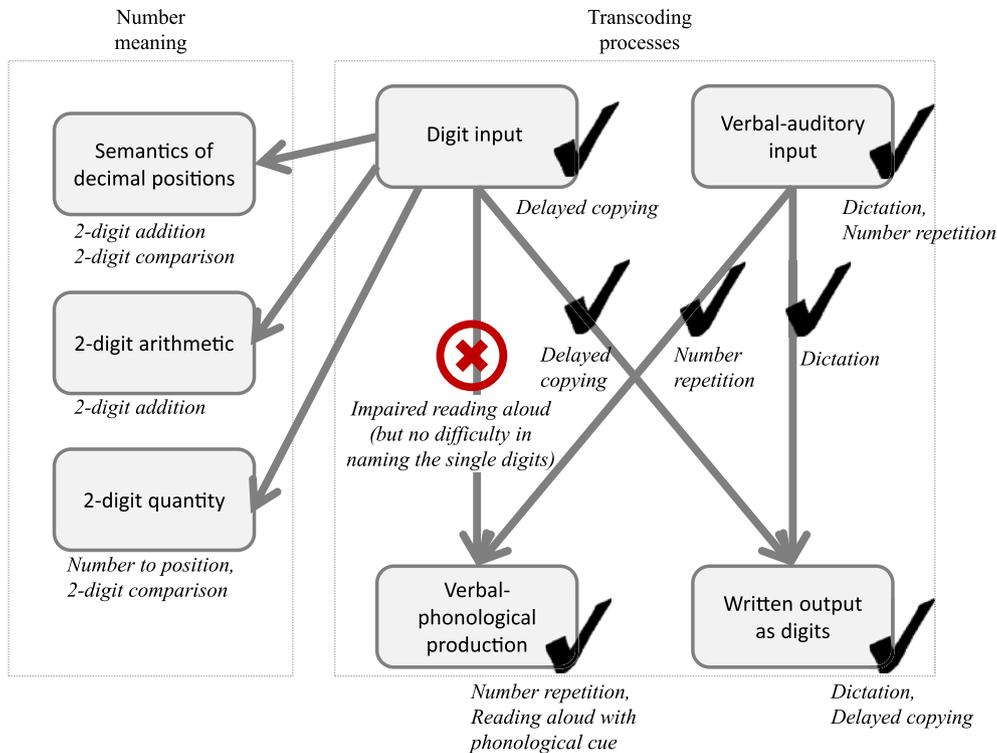


Fig. 1 – An overview of the numerical processes that were tested in ZN. The italic text denotes the tasks that indicate which processes are spared and which are impaired.

a phoneme-by-phoneme strategy for repeating numbers. Last, several studies of number production mechanisms render the sublexical repetition hypothesis unlikely (Bencini et al., 2011; Cohen, Verstichel, & Dehaene, 1997; Dotan & Friedmann, 2010, in press): these studies investigated aphasic patients with phonological deficits and showed that even in a very late stage of speech production, the phonological output buffer, whole number words are processed as atomic phonological units rather than as separate phonemes.

The above experiments show that ZN has a deficit in one of the modules along the Arabic-to-verbal transcoding route, which selectively affects his ability to say two-digit numbers, while sparing single-digit numbers. The deficit is in a stage later than the Arabic number input modules and earlier than the phonological production stages, i.e., the deficit is in the process of converting the digits to number words (this pathway is marked in Fig. 1 with ⊗). Furthermore, the deficit is syntactic as it affects only two-digit numbers (and longer numbers too, although they were not systematically explored in this study) whereas single digits are spared.

At this point we know that ZN's deficit is in a syntactic module in the process that converts multi-digit Arabic numerals into number words. We can think of this process as involving two stages – a “core” stage that converts numbers from a sequence of digits to a set of abstract identities of number words (Cohen & Dehaene, 1991; Dotan & Friedmann, 2007, 2010, in press; McCloskey, Sokol, Caramazza, & Goodman-Schulman, 1990; McCloskey et al., 1986; Sokol & McCloskey, 1988), and a subsequent stage that transfers this information to the morpho-phonological production modules. Because his number repetition indicates that the source of his

number deficit is not the phonological output itself, we can conclude that the latest possible locus of deficit is in the module that sends the output of the conversion process, namely, the abstract identities of number words, to the phonological modules of lexical retrieval.

Although ZN's deficit is in number syntax, our findings rule out the extreme possibility that he has lost all syntactic abilities: in both number reading tasks (as numbers and as single digits) he never said the unit digit before the decade digit. Thus, the relative order of the two digits, which is one kind of syntactic information, was spared. ZN's good dictation of two-digit numbers showed that his verbal-to-digit syntactic processing was also spared. As the next section will show, these were not the only kind of spared syntactic abilities.

4. Assessment of number comprehension

We have now reached the main question of this study: we saw that ZN cannot convert two-digit Arabic numbers to the phonological form of number words. Can he still understand two-digit numbers?

We were interested in two general questions about his comprehension of numbers – can he understand two-digit numbers, and what is the nature of the processes he uses to understand numbers. More specifically, the first question we will ask is whether he can apply the process that converts the relative positions of the digits into their abstract decimal roles as decades and units. This question was assessed using a two-digit addition task and a number comparison task (see the left column in Fig. 1).

The second question relates to the way he processes the quantity corresponding with two-digit numbers. Two-digit numbers can be encoded as decomposed decade and unit quantities (Meyerhoff, Moeller, Debus, & Nuerk, 2012; Moeller, Klein, Nuerk, & Willmes, 2013; Moeller, Nuerk, & Willmes, 2009; Nuerk & Willmes, 2005) but they can also be encoded by combining the tens and units values into an appropriate overall holistic quantity (Brybaert, 1995; Dehaene et al., 1990; Dotan & Dehaene, 2013; Reynvoet & Brybaert, 1999). Does ZN encode a two-digit number as a holistic quantity? Such a holistic encoding would unequivocally show that he not only managed to categorize the two digits into their two decimal roles and understand the quantity represented by each digit, but he was also able to reach a combined quantity by evaluating the two digits in correct proportions. This second question was assessed using the number comparison task and an Arabic number-to-position mapping task.

4.1. Two-digit addition

The two-digit addition task examined ZN's ability to assign the two digits into their decimal roles as decades and units, and apply the procedures required to add them. He was presented with written exercises in which he added single-digit numbers to other single-digit numbers or to two-digit numbers. The exercises were always presented using Arabic numbers and ZN answered in writing. He was shown three single-digit additions ($X + Y$) with a single-digit result, four round-number additions ($X0 + Y$ or $X00 + Y$), and eight two-digit additions ($XY + Z$), five of which required carry procedure. His performance was flawless.

This performance with written answers is markedly different from his performance when he was required to give oral responses to two-digit additions. He could not answer orally any of the 3 two-digit addition exercises presented to him ($XY + Z$), and responded by saying only the (correct) unit digit of the result. He performed flawlessly, however, with single digit additions ($X + Y$; 3/3 correct), and had only one error in 4 round-number additions.

Thus, in spite of his deficit in verbal production of multi-digit numbers, ZN can still perform two-digit addition, even when a carry procedure is required, as long as he is not required to produce the result orally.

4.2. Two-digit comparison

A common task to examine holistic processing of numbers is two-digit comparison. In the task that we used, ZN was asked to decide in each trial whether the two-digit number presented on screen is smaller or larger than 55 ("the standard"). Number comparison tasks involve comparison of magnitudes, and reaction times decrease when the target-standard distance increases (Dehaene et al., 1990; Hinrichs & Novick, 1982; Moyer & Landauer, 1967; Nuerk & Willmes, 2005). In the present task, if the participant uses two-digit holistic quantity, we should observe a continuous distance effect (Dehaene et al., 1990).

4.2.1. Method

A two-digit number was presented on screen in each trial. ZN was asked to compare each number as quickly as possible to a

fixed reference number of 55; he pressed the "Z" key with his left hand to respond "smaller than 55", or the "." key with his right hand to respond "larger than 55". Each of the numbers from 31 to 79, except 55, was shown 4 times. The 192 trials were presented in random order. The experiment was implemented using PsychToolbox with Matlab R2012a on a Macbook Pro laptop with a 13" monitor. The numbers were presented in the center of the screen in black font on gray background. The digits were 2 cm high.

4.2.2. Results

ZN had 2.6% errors in this task (5 errors), and these trials were excluded from the analyses. His RT was 877 ± 273 msec, with no significant difference between hands (right: 909 ± 346 msec; left: 844 ± 161 msec, $t(185) = 1.66$, two-tailed $p = .10$).²

To analyze the effect of target-standard distance, the trials were grouped into 3 groups by their distance from 55 (distances of 1–8, 9–16, or 17–23). The RT was submitted to ANOVA with the distance group and response type (smaller/larger than 55) as factors. Both factors had a significant main effect [distance group: $F(2,181) = 4.2$, two-tailed $p = .02$; response type: $F(1,181) = 2.76$, one-tailed $p = .05$] and there was no interaction [$F(2,181) = 1.11$, $p = .33$]. The mean RTs of the three distance groups were 954, 865, and 814 msec respectively, and the linear contrast was significant [$F(1,181) = 8.18$, $p = .005$], which confirms the predicted distance effect.

The RTs were then submitted to a regression analysis. 14 outlier trials were removed (a trial was defined as an outlier with respect to the four trials of the same target number, if removing this trial decreased the standard deviation to 33% or less). The predictors were $\log(\text{absolute distance between target and standard})$, the response type (1 or -1), and the product of these two (to assess interaction). Only $\log(\text{distance})$ had a significant effect ($p = .001$), confirming again the distance effect. The two other predictors were not significant ($p > .23$).

To study the contribution of the unit digit to the distance effect, we used two regression analyses introduced by Dehaene et al. (1990). In the first analysis, the predictors were LogDiz , which represents the decade distance, and Dunit , which assesses the unit digit contribution.³ Both predictors were significant ($p < .04$), which shows that both digits affected the comparison. The second analysis was run only on trials outside the standard's decade. The predictors were $\log(\text{absolute distance between target and standard})$, response type, their product, and Dunit . Only $\log(\text{distance})$ had significant contribution ($p < .02$, and $p \geq .13$ for the other predictors), showing that the holistic distance is a sufficient predictor of the distance effect, and that the decomposed unit makes no additional observable contribution to the RT.

² The variance in right-hand responses was larger than in the left hand [$F(90,95) = 4.64$, $p < .0001$], perhaps as a result of the left-hemisphere brain damage.

³ Let D_s , U_s , D_t , and U_t be respectively the decades and units digits of the standard and the target ($D_s = U_s = 5$). LogDiz is the logarithm of $1 + |D_t - D_s|$. Dunit equals zero for targets within the standard's decade (i.e., when $D_t = D_s$). Outside the standard's decade, Dunit equals $U_t - 4.5$ for targets smaller than the standard and $4.5 - U_t$ for targets larger than the standard.

4.2.3. Discussion of the two-digit comparison task

ZN's high accuracy on this task clearly indicates that he was able to understand two-digit numbers and assign the digits to their appropriate decimal roles as decades and units. Furthermore, his performance is in accord with the assumption that he used holistic encoding of the two-digit quantities, as we observed a target-standard distance effect that extended beyond the standard's decade, with no additional contribution of the unit digit.

4.3. Number-to-position

The number-to-position task is another common paradigm to investigate quantity representation. In this task, an individual is shown a target number and is asked to mark the corresponding position on an unmarked number line. The way individuals map numbers to positions reflects to some extent the structure of the mental number line, and hence of the quantity representation (Barth & Paladino, 2011; Berteletti, Lucangeli, Piazza, Dehaene, & Zorzi, 2010; Booth & Siegler, 2006; Cappelletti, Kopelman, Morton, & Butterworth, 2005; Dotan & Dehaene, 2013; Siegler & Booth, 2004; Siegler & Opfer, 2003; Von Aster, 2000). Here, ZN performed the number-to-position task on an iPad tablet computer, while his finger trajectory was tracked throughout each trial – from the moment the target number was shown on screen until ZN marked with his finger a position on the number line. This trajectory tracking paradigm taps the two-digit quantity representation and the way it evolves during a trial (Dotan & Dehaene, 2013).

4.3.1. Method

The number line was always presented on top of the screen (see Fig. 2a). Each trial began when ZN touched a specific point at the middle-bottom of the screen. As soon as the finger started moving upwards, the target number appeared above the middle of the number line, and ZN dragged his finger upwards until it touched the number line. At this time the target number disappeared and a feedback arrow indicated the responded position. Each of the 41 target numbers between 0 and 40 was presented four times, all in random order. Four trials were excluded because ZN touched the screen with multiple fingers or started the trial with a sideways rather than upwards movement. The target numbers of these trials were re-presented later, so the total number of valid trials was still 164. The experiment software tracked the full finger trajectory per trial.

ZN's results were compared with a control group of 15 right-handed individuals matched for age ($70; 7 \pm 4; 2$, from 66; 6 to 79; 7), language (native Hebrew speakers), education (BA or MA degree), and occupation (they all worked, like ZN, in number-oriented jobs: 9 engineers, 3 math teachers in high or junior high schools, 2 economists, and one accountant).

4.3.2. Results

ZN's finger movement was 1320 ± 220 msec from target onset to reaching the number line, which is similar to the control participants (1290 ± 210 msec, Crawford and Garthwaite (2002) and Crawford and Howell (1998) $t(14) = .14$, one-tailed $p = .45$). Fig. 2b shows ZN's finger trajectories (for each target number, a median trajectory was calculated by re-sampling the raw

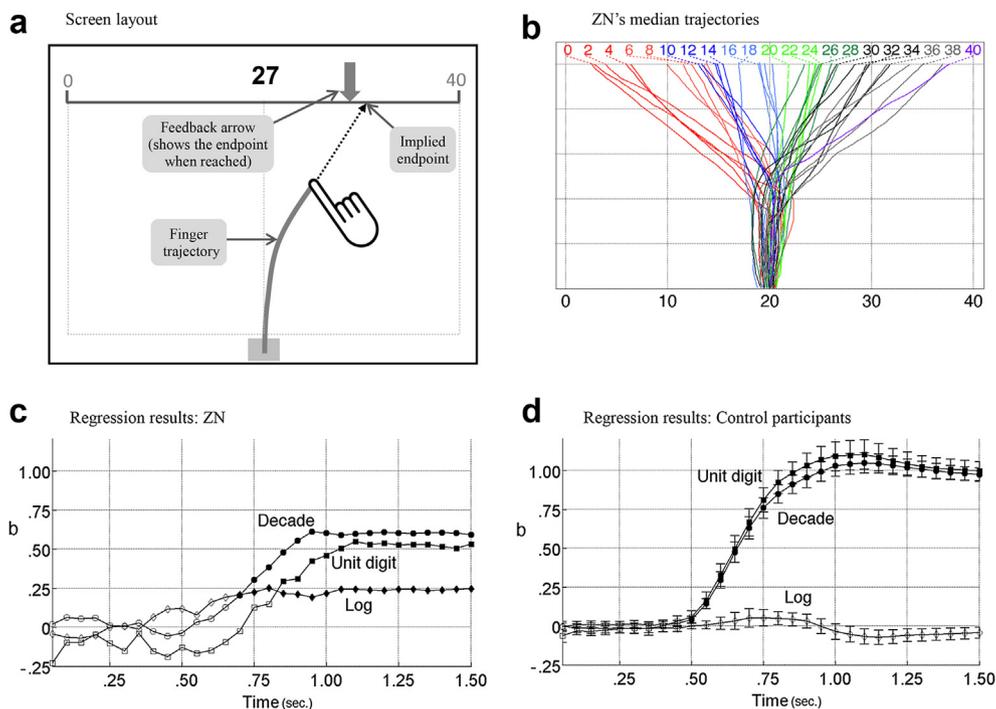


Fig. 2 – (a) Task and screen layout. ZN was asked to position a 2-digit number on a horizontal line that extended from 0 to 40. He initiated a trial by placing his finger on the bottom rectangle. The target appeared when he started moving his finger upward. (b) ZN's performance: median trajectories per target. (c–d) The b values of the regressions that capture the results of the number-to-position task. Each group of 3 vertically-aligned points represents a single regression of a specific post-stimulus-onset time ($t = 0$ is the stimulus onset). Significant b values ($p \leq .05$) are represented by black markers.

trajectories into equally time points and finding the median coordinate per time point).

The quantity representation in this task can be investigated by finding which factors govern the finger's horizontal movement. This was done using a regression analysis. The dependent variable was the trajectory endpoints (the positions marked by ZN on the number line), and there were three predictors. The first two predictors account for the linear quantity representation: the target number's decade (0, 10, 20, 30, or 40) and the unit digit. The two digits were entered as separate predictors to account for the possibility that the contributions of the decomposed decade and unit quantities deviate from a 1:10 ratio. The third predictor was $\log(\text{target} + 1)$, linearly rescaled to the 0–40 range. This predictor taps holistic-logarithmic representation of quantity (for a detailed explanation of this regression analysis method and of the choice of predictors, see [Dotan & Dehaene, 2013](#)). The regression showed significant contribution of the all three predictors ($p < .001$).⁴

A similar regression analysis was performed to assess how ZN's quantity representation evolves throughout a trial (from stimulus onset until he touches the number line). One regression was run per post-stimulus-onset time point, in 50 msec intervals. The same three predictors were used (decade, unit, log) and the dependent variable was the implied endpoint – the position on the number line that the finger would hit if it keeps moving in its current direction θ_t . This θ_t was defined as the direction vector between the finger x , y coordinates at times $t - 50$ msec and t . The implied endpoint was also cropped to the range $[-2, 42]$ and was undefined when the finger moved sideways ($|\theta| > 80^\circ$). The regression was also run for each of the control participants, and the significance of each b value in the control group (per predictor and time point) was assessed by comparing the group's b values with 0 using t -test. One-tailed p values were used for average(b) > 0 , and two-tailed p values for average(b) < 0 .

This sequence of regressions ([Fig. 2c](#)) showed that ZN had significant contributions of the decade predictor from 700 msec post-stimulus-onset and in all subsequent time points, of the units from 850 msec, and of the log from 650 msec. The control group ([Fig. 2d](#)) showed an earlier effect of the decade (from 500 msec) and the units digit (from 550 msec), and no significant group-level log effect. We now turn to a detailed analysis and comparison of these effects.

4.3.2.1. ZN ENCODES HOLISTIC TWO-DIGIT QUANTITIES. The existence of a significant logarithmic factor clearly shows that ZN represented a holistic quantity that integrated the decade and unit values of the target number. This is because the logarithmic function cannot be represented as a linear combination of the decade and unit quantities, so a logarithmic factor

⁴ In a previous study, which investigated healthy participants, the regression analyses discovered a fourth significant predictor that reflects a spatial aiming strategy in the late trajectory parts (“the spatial reference points” effect, [Dotan & Dehaene, 2013](#), section 3.2.6). However, in ZN's data this predictor had no significant effect on the trajectory endpoints ($b < .001$, $p > .93$), nor was it significant in the trajectory analysis described in the next paragraph ($p > .12$ in all time points). Thus, the present study did not use this predictor.

in the regression necessarily reflects a log or log-like function of the whole quantity. Importantly, there was a time window of 100 msec, starting in 650 msec, in which the log predictor was significant but the linear predictors (decade and unit) were not yet significant. This indicates that the holistic-logarithmic quantity representation preceded the linear representation.

We compared ZN's performance pattern with the group of healthy control participants using the three-predictor regression model described above. ZN's b value of the log predictor was higher than the control group in 250 msec and in all subsequent time points, and this difference was marginally significant in 1250 msec and in the subsequent time points ([Crawford and Garthwaite \(2002\)](#) $t \geq 1.83$, two-tailed $p \leq .1$ per time point). A per-participant analysis showed that 4 control participants had a significant log effect in 2 or more time points. ZN's log effect remained quite stable throughout the trajectory and was observable even in the endpoints. This pattern is different from the control participants, for whom the non-significant logarithmic trend was clearly transient (see [Fig. 2d](#)).⁵

We examined and excluded an alternative explanation according to which ZN employed a decomposed quantity representation of each of the digits using a logarithmic quantity scale. According to such an alternative explanation, the log factor in [Fig. 2c](#) is an artifact of the correlation between the $\log(\text{target} + 1)$ predictor, which we used in the regression, and the factors that allegedly governed ZN's hand movement: some linear combination of $\log(\text{decade})$ and $\log(\text{unit-digit})$. To rule out this possibility, another regression analysis was run: the dependent variable was still the implied endpoint, but the logarithms of the decade and unit were added as two new predictors on top of the decade digit, the unit digit, and $\log(\text{target} + 1)$. One such regression was run per post-stimulus-onset time point, in 50 msec intervals. The results showed significant contributions of $\log(\text{target} + 1)$ in 800 msec and in all time points from 900 msec ($p < .05$). Importantly, there was no time point in which any of the single-digit logarithms made a significant contribution. In fact, their regression b values were negative in 900 msec and in all subsequent time points.

Another alternative explanation that we ruled out was the possibility that the log effect results from faster encoding of smaller quantities. Faster processing of small-target trials would make their initial finger trajectories farther apart from each other than the trajectories of larger target numbers, and this gives rise to the log effect. To neutralize this differential quantity encoding speed, we aligned trajectories by the time point of the first significant horizontal finger movement. First, the horizontal velocity along each trajectory was calculated by

⁵ ZN's log effect was also compared with another control group of 21 younger participants, reported in [Dotan and Dehaene \(2013\)](#). This comparison too showed that ZN's performance pattern was no less logarithmic than the control group's – in fact, his $b[\log]$ was larger than the $b[\log]$ of this control group in 600 msec and in all subsequent time points, and this difference was significant from 850 msec and onwards ([Crawford & Garthwaite's \(2002\)](#) $t \geq 2.28$, two-tailed $p \leq .04$; and from 1000 msec, $t \geq 4.05$, $p < .001$). The log effect of the younger control participants was also transient, like the older control group (and unlike ZN).

smoothing the x coordinates using a 30 msec running average and then deriving them. To determine the horizontal movement onset per trial, we first looked for a significant peak of the x velocity profile – i.e., a velocity exceeding the first percentile of the participant's velocity distribution on the first 300 msec of all trials. We then found the last time point where the x velocity remained lower than 5% of this peak velocity. We excluded trials in which no peak velocity was high enough, trials in which the peak velocity was in the wrong right/left direction, and trials where the above 5% criterion was never met, or was met earlier than 300 msec. After finding each trial's horizontal velocity onset time, the decade-unit-log regression was rerun while aligning ZN's trajectory data to this onset time. Even in these regressions, which neutralize possible differences in velocity onset per trial (and per target), ZN still showed a logarithmic effect ($b[\log] > .12$, one-tailed $p < .05$, in all time points from 700 msec post-velocity-onset, and $p < .07$ from 500 msec), thereby refuting the differential velocity onset as an alternative explanation.

The results show unequivocally that ZN used holistic encoding of two-digit quantities, and that this holistic encoding was not impaired in comparison to the control group.

4.3.2.2. DECOMPOSED LINEAR QUANTITY ENCODING. Fig. 2c shows that ZN's effect of the unit digit seems slightly delayed with respect to the decade digit. This difference was statistically assessed by modifying the predictors in the above per time point regression analysis into $\log(\text{target} + 1)$, the target number N_{0-40} , and the unit digit U . In this new set of regressions, the predictor U captures situations in which the relative contributions of the decade and unit digits deviate from a strict 1:10 ratio. Such deviation was indeed found: the unit digit predictor's b value ($b[U]$) was smaller than zero in all time points, and this difference was significant in a certain time window (two-tailed $p < .05$ in 800 and 900 msec; and $p < .1$ from 650 msec to 950 msec except in 750 msec). These results suggest that ZN was processing the decade and unit digits in decomposed and possibly serial manner.

ZN's delayed processing of the unit digit was not statistically different from the control group: comparing his $b[U]$ in the $\log + \text{target} + \text{unit}$ regression with the control participants showed no significant difference in any time point (even when assuming that his $b[U]$ should be smaller than the controls' and consequently using one-tail p values, only a marginally significant difference was found in only 3 time points – 800, 900, and 950 msec – Crawford and Garthwaite (2002) $t \leq -1.49$, $p < .1$). A per-participant analysis of the control group showed that three participants also showed a significant $b[U] < 0$ in two or more time points. Thus, even if ZN's processing of the decade and unit digits appeared slightly more sequential than the control group's, this difference was very small.

4.3.2.3. ACCURACY. ZN's endpoint error – the absolute difference between the judged endpoint and the correct target position – was 3.13 ± 2.43 (using the 0–40 scale). This is less accurate than the control participants, whose mean endpoint error was $1.94 \pm .53$ [Crawford and Garthwaite (2002) $t(14) = 2.17$, $p = .02$]. ZN's endpoint errors were not correlated with the target number ($r = -.05$, $p = .53$), nor was there

another, non-linear dependency between the target number and the endpoint error [one-way ANOVA, $F(40,123) = 1.33$, $p = .12$].

4.3.3. Discussion of the number-to-position task

ZN's performance in this task showed that he encoded two-digit quantities holistically. There was no evidence to suggest that the holistic encoding was impaired with respect to healthy participants – in fact, the holistic-logarithmic trend in ZN's result was even slightly higher than in the control group. Given ZN's severe syntactic deficit in converting two-digit numbers from digit to verbal representation, we can reach the most important conclusion in this study: constructing the holistic quantity was performed successfully, independently of the impairment in digit-to-verbal conversion. Furthermore, an analysis of the linear factors in this task suggests that ZN's ability to process the decade and unit digits in parallel was comparable with that of the control participants, or only slightly worse.

5. General discussion

This study presented the case of ZN, an aphasic patient who has a selective syntactic deficit in converting two-digit numbers from digit representation to verbal-phonological representation. ZN can read aloud single digits but he has great difficulty in reading aloud two-digit Arabic numbers using a valid decade + unit syntactic structure. This difficulty also affects longer numbers (with 3 digits or more), although the present study systematically investigated only single-digit and two-digit numbers. A detailed neuropsychological examination showed that ZN's deficit is neither in the Arabic input nor in the phonological output modules, because he could copy multi-digit numbers, write them to dictation, and repeat them. His syntactic deficit therefore lies in the central process that converts the digits into a structured sequence of abstract identities of number words (the *number word frame*, Cohen & Dehaene, 1991; Dotan & Friedmann, 2007, 2010, *in press*), or in a subsequent stage that uses these abstract identities to access the phonological production modules. This deficit is not a global deficit in processing number syntax: ZN has intact syntactic processing in the opposite pathway – verbal to digit representation – as demonstrated by his good performance in number dictation. This dissociation between digit-to-verbal and verbal-to-digit syntax is in line with previous studies (Cipolotti, 1995).

In spite of his deficit in digit to verbal number conversion, ZN showed spared number comprehension and spared number syntax abilities in several ways. First, he is able to add two-digit numbers with single-digit numbers, even when the addition exercise requires carry operation, as long as verbal output is not required. This shows that he understands the base-10 system, can assign the digits to their decimal roles as decades and units, and can carry out the addition procedure. This finding extends previous studies showing that multi-digit addition does not depend on phonological (Klessinger et al., 2012; Varley et al., 2005) and orthographic (Varley et al., 2005) representations of verbal numbers: whereas those studies showed that addition does not depend on

phonological encoding of the numbers, we showed that addition does not depend even on an earlier stage – a syntactic module involved in digit-to-verbal transcoding. In this sense our conclusions resemble Brysbaert et al.'s (1998), who showed that addition is unaffected by the syntactic structure of verbal numbers in a certain language. However, whereas Brysbaert et al.'s conclusion rested on a null effect of language in nonverbal calculation, we managed to show a strict dissociation between spared addition and impaired syntactic processing.

Crucially, ZN's spared comprehension and syntax was also shown by his ability to encode two-digit numbers as holistic quantities. This was demonstrated by the finding of a continuous two-digit distance effect in the two-digit number comparison task, and by the finding of a logarithmic factor in the two-digit number-to-position mapping task. His good performance in these tasks also demonstrated his ability to assign digits to decimal roles. Fig. 3 illustrates these conclusions.

These findings lead to interesting conclusions regarding the specificity of the modules that process number syntax. Multi-digit Arabic numbers require syntactic processing when converted to verbal number words, when converted to quantities, and when manipulated in addition exercises. Our results unequivocally show that certain syntactic functions – assigning digits to their decimal roles and converting two-digit Arabic numbers to holistic quantities – are dissociable from at least some of the syntactic processes involved in digit-to-verbal transcoding, because these syntactic functions can be successfully performed even when one of the syntactic digit-to-verbal transcoding processes is impaired.

These results are in line with several previous studies that dissociated between Arabic number comprehension and Arabic-to-verbal transcoding. Several previous patients showed impairments of digit-to-word conversion with spared number comprehension (Cohen & Dehaene, 1995, 2000; Cohen, Dehaene, & Verstichel, 1994). Other studies

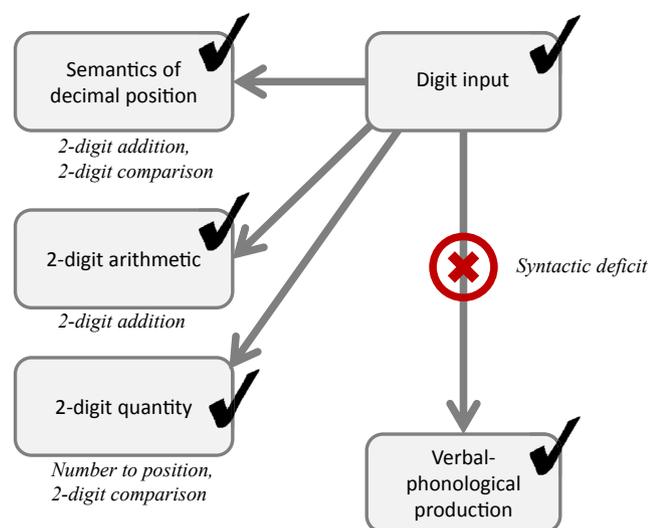


Fig. 3 – In spite of ZN's syntactic deficit in converting numbers in Arabic notation to a verbal representation, he can assign the digits to their decimal roles, encode two-digit holistic quantities, and perform two-digit additions.

specifically reported patients with a syntactic deficit in digit-to-word conversion (Cipolotti & Butterworth, 1995; patient SAM; Cipolotti, 1995; patient SF) who could perform certain number comprehension tasks – number comparison (both patients), multi-digit comparison (SF), and two-digit addition (SAM). The syntactic deficit of SAM and SF still allowed them to assign digits to decimal roles. The present findings replicate and extend these results, particularly using the number-to-line task to demonstrate a fine-grained preservation of the encoding of two-digit numbers as holistic quantities in patient ZN.

Taken together, such neuropsychological cases indicate that the syntactic processes involved in converting digits to words and digits to quantities are at least partially separate, and that several aspects of two-digit number comprehension can be achieved without transcoding the number to its verbal representation. This conclusion fits with several other findings that dissociated language syntax from several aspects of syntax-dependent mathematical processing (Brysbaert et al., 1998; Maruyama et al., 2012; Monti et al., 2012; Varley et al., 2005). Taken together, this body of evidence weakens the hypothesis that a single global mechanism underlies all kinds of syntactic processes (Hauser et al., 2002; Houdé & Tzourio-Mazoyer, 2003) and promotes a view of several, distributed syntactic processes.

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