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Are our brains wired for math?

by Jim Holt

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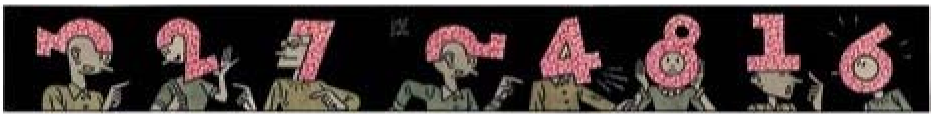

According to Stanislas Dehaene, humans have an inbuilt "number sense" capable of some basic calculations and estimates. The problems start when we learn mathematics and have to perform procedures that are anythin but instinctive.

One morning in September, 1989, a former sales representative in his mid-forties entered an examination room with Stanislas Dehaene, a young neuroscientist based in Paris. Three years earlier, the man, whom researchers came to refer to as Mr. N, had sustained a brain hemorrhage that left him with an enormous lesion in the rear half of his left hemisphere. He suffered from severe handicaps: his right arm was in a sling; he couldn't read; and his speech was painfully slow. He had once been married, with two daughters, but was now incapable of leading an independent life and lived with his elderly parents. Dehaene had been invited to see him because his impairments included severe acalculia, a general term for any one of several deficits in number processing. When asked to add 2 and 2, he answered "three." He could still count and recite a sequence like 2, 4, 6, 8, but he was incapable of counting downward from 9, differentiating odd and even numbers, or recognizing the numeral 5 when it was flashed in front of him.

To Dehaene, these impairments were less interesting than the fragmentary capabilities Mr. N had managed to retain. When he was shown the numeral 5 for a few seconds, he knew it was a numeral rather than a letter and, by counting up from 1 until he got to the right integer, he eventually identified it as a 5. He did the same thing when asked the age of his seven-year-old daughter. In the 1997 book "The Number Sense," Dehaene wrote, "He appears to know right from the start what quantities he wishes to express, but reciting the number series seems to be his only means of retrieving the corresponding word."

Dehaene also noticed that although Mr. N could no longer read, he sometimes had an approximate sense of words that were flashed in front of him; when he was shown the word "ham," he said, "It's some kind of meat." Dehaene decided to see if Mr. N still had a similar sense of number. He showed him the numerals 7 and 8. Mr. N was able to answer quickly that 8 was the larger number—far more quickly than if he had had to identify them by counting up to the right quantities. He could also judge whether various numbers were bigger or smaller than 55, slipping up only when they were very close to 55. Dehaene dubbed Mr. N "the Approximate Man." The Approximate Man lived in a world where a year comprised "about 350 days" and an hour "about fifty minutes," where there were five seasons, and where a dozen eggs amounted to "six or ten." Dehaene asked him to add 2 and 2 several times and received answers ranging from three to five. But, he noted, "he never offers a result as absurd as 9."

In cognitive science, incidents of brain damage are nature's experiments. If a lesion knocks out one ability but leaves another intact, it is evidence that they are wired into different neural circuits. In this instance, Dehaene theorized that our ability to learn sophisticated mathematical procedures resided in an entirely different part of the brain from a rougher quantitative sense. Over the decades, evidence concerning cognitive deficits in brain-damaged patients has accumulated, and researchers have concluded that we have a sense of number that is independent of language, memory, and reasoning in general. Within neuroscience, numerical cognition has emerged as a vibrant field, and Dehaene, now in his early forties, has become one of its foremost researchers. His work is "completely pioneering," Susan Carey, a psychology professor at Harvard who has studied numerical cognition, told me. "If you want to make sure the math that children are learning is meaningful, you have to know something about how the brain represents number at the kind of level that Stan is trying to understand."

Dehaene has spent most of his career plotting the contours of our number sense and puzzling over which aspects of our mathematical ability are innate and which are learned, and how the two systems overlap and affect each other. He has approached the problem from every imaginable angle. Working with colleagues both in France and in the United States, he has carried out experiments that probe the way numbers are coded in our minds. He has studied the numerical abilities of animals, of Amazon tribespeople, of top French mathematics students. He has used brain-scanning technology to investigate precisely where in the folds and crevices of the cerebral cortex our numerical faculties are nestled. And he has weighed the extent to which some languages make numbers more difficult than others. His work raises crucial issues about the way mathematics is taught. In Dehaene's view, we are all born with an evolutionarily ancient mathematical instinct. To become numerate, children must capitalize on this instinct, but they must also unlearn certain tendencies that were helpful to our primate ancestors but that clash with skills needed today. And some societies are evidently better than others at getting kids to do this. In both France and the United States, mathematics education is often felt to be in a state of crisis. The math skills of American children fare poorly in comparison with those of their peers in countries like Singapore, South Korea, and Japan. Fixing this state of affairs means grappling with the question that has taken up much of Dehaene's career: What is it about the brain that makes numbers sometimes so easy and sometimes so hard?

Dehaene's own gifts as a mathematician are considerable. Born in 1965, he grew up in Roubaix, a medium-sized industrial city near France's border with Belgium. (His surname is Flemish.) His father, a pediatrician, was among the first to study fetal alcohol syndrome. As a teen-ager, Dehaene developed what he calls a "passion" for mathematics, and he attended the École Normale Supérieure in Paris, the training ground for France's scholarly élite. Dehaene's own interests tended toward computer modelling and artificial intelligence. He was drawn to brain science after reading, at the age of eighteen, the 1983 book "Neuronal Man," by Jean-Pierre Changeux, France's most distinguished neurobiologist. Changeux's approach to the brain held out the tantalizing possibility of reconciling psychology with neuroscience. Dehaene met Changeux and began to work with him on abstract models of thinking and memory. He also linked up with the cognitive scientist Jacques Mehler. It was in Mehler's lab that he met his future wife, Ghislaine Lambertz, a researcher in infant cognitive psychology.

By "pure luck," Dehaene recalls, Mehler happened to be doing research on how numbers are understood. This led to Dehaene's first encounter with what he came to characterize as "the number sense." Dehaene's work centered on an apparently simple question: How do we know whether numbers are bigger or smaller than one another? If you are asked to choose which of a pair of Arabic numerals—4 and 7, say—stands for the bigger number, you respond "seven" in a split second, and one might think that any two digits could be compared in the same very brief period of time. Yet in Dehaene's experiments, while subjects answered quickly and accurately when the digits were far apart, like 2 and 9, they slowed down when the digits were closer together, like 5 and 6. Performance also got worse as the digits grew larger: 2 and 3 were much easier to compare than 7 and 8. When Dehaene tested some of the best mathematics students at the École Normale, the students were amazed to find themselves slowing down and making errors when asked whether 8 or 9 was the larger number.

Dehaene conjectured that, when we see numerals or hear number words, our brains automatically map them onto a number line that grows increasingly fuzzy above 3 or 4. He found that no amount of training can change this. "It is a basic structural property of how our brains represent number, not just a lack of facility," he told me.

In 1987, while Dehaene was still a student in Paris, the American cognitive psychologist Michael Posner and colleagues at Washington University in St. Louis published a pioneering paper in the journal *Nature*. Using a scanning technique that can track the flow of blood in the brain, Posner's team had detailed how different areas became active in language processing. Their research was a revelation for Dehaene. "I remember very well sitting and reading this paper, and then debating it with Jacques Mehler, my Ph.D. adviser," he told me. Mehler, whose focus was on determining the abstract organization of cognitive functions, didn't see the point of trying to locate precisely where in the brain things happened, but Dehaene wanted to "bridge the gap," as he put it, between psychology and neurobiology, to find out exactly how the functions of the mind—thought, perception, feeling, will—are realized in the gelatinous three-pound lump of matter in our skulls. Now, thanks to new technologies, it was finally possible to create pictures, however crude, of the brain in the act of thinking. So, after receiving his doctorate, he spent two years studying brain scanning with Posner, who was by then at the University of Oregon, in Eugene. "It was very strange to find that some of the most exciting results of the budding cognitive-neuroscience field were coming out of this small place—the only place where I ever saw sixty-year-old hippies sitting around in tie-dyed shirts!" he said.

Dehaene is a compact, attractive, and genial man; he dresses casually, wears fashionable glasses, and has a glabrous dome of a head, which he protects from the elements with a *chapeau de cowboy*. When I visited him recently, he had just moved into a new laboratory, known as NeuroSpin, on the campus of a national center for nuclear-energy research, a dozen or so miles southwest of Paris. The building, which was completed a year ago, is a modernist composition in glass and metal filled with the ambient hums and whirs and whooshes of brain-scanning equipment, much of which was still being assembled. A series of arches ran along one wall in the form of a giant sine wave; behind each was a concrete vault built to house a liquid-helium-cooled superconducting electromagnet. (In brain imaging, the more powerful the magnetic field, the sharper the picture.) The new brain scanners are expected to show the human cerebral anatomy at a level of detail never before seen, and may reveal subtle anomalies in the brains of people with dyslexia and with dyscalculia, a crippling deficiency in dealing with numbers which, researchers suspect, may be as widespread as dyslexia. One of the scanners was already up and running. "You don't wear a pacemaker or anything, do you?" Dehaene asked me as we entered a room where two researchers were fiddling with controls. Although the scanner was built to accommodate humans, inside, I could see from the monitor, was a brown rat. Researchers were looking at how its brain reacted to various odors, which were puffed in every so often. Then Dehaene led me upstairs to a spacious gallery where the brain scientists working at NeuroSpin are expected to congregate and share ideas. At the moment, it was empty. "We're hoping for a coffee machine," he said.

Dehaene has become a scanning virtuoso. On returning to France after his time with Posner, he pressed on with the use of imaging technologies to study how the mind processes numbers. The existence of an evolved number ability had long been hypothesized, based on research with animals and infants, and evidence from brain-damaged patients gave clues to where in the brain it might be found. Dehaene set about localizing this facility more precisely and describing its architecture. "In one experiment I particularly liked," he recalled, "we tried to map the whole parietal lobe in a half hour, by having the subject perform functions like moving the eyes and hands, pointing with fingers, grasping an object, engaging in various language tasks, and, of course, making small calculations, like thirteen minus four. We found there was a beautiful geometrical organization to the areas that were activated. The eye movements were at the back, the hand movements were in the middle, grasping was in the front, and so on. And right in the middle, we were able to confirm, was an area that cared about number."

The number area lies deep within a fold in the parietal lobe called the intraparietal sulcus (just behind the crown of the head). But it isn't easy to tell what the neurons there are actually doing. Brain imaging, for all the sophistication of its technology, yields a fairly crude picture of what's going on inside the skull, and the same spot in the brain might light up for two tasks even though different neurons are involved. "Some people believe that psychology is just being replaced by brain imaging, but I don't think that's the case at all," Dehaene said. "We need psychology to refine our idea of what the imagery is going to show us. That's why we do behavioral experiments, see patients. It's the confrontation of all these different methods that creates knowledge."

Dehaene has been able to bring together the experimental and the theoretical sides of his quest, and, on at least one occasion, he has even theorized the existence of a neurological feature whose presence was later confirmed by other researchers. In the early nineteen-nineties, working with Jean-Pierre Changeux, he set out to create a computer model to simulate the way humans and some animals estimate at a glance the number of objects in their environment. In the case of very small numbers, this estimate can be made with almost perfect accuracy, an ability known as "subitizing" (from the Latin word *subitus*, meaning "sudden"). Some psychologists think that subitizing is merely rapid, unconscious counting, but others, Dehaene included, believe that our minds perceive up to three or four objects all at once, without having to mentally "spotlight" them one by one. Getting the computer model to subitize the way humans and animals did was possible, he found, only if he built in "number neurons" tuned to fire with maximum intensity in response to a specific number of objects. His model had, for example, a special four neuron that got particularly excited when the computer was presented with four objects. The model's number neurons were pure theory, but almost a decade later two teams of researchers discovered what seemed to be the real item, in the brains of macaque monkeys that had been trained to do number tasks. The number neurons fired precisely the way Dehaene's model predicted—a vindication of theoretical psychology. "Basically, we can derive the behavioral properties of these neurons from first principles," he told me. "Psychology has become a little more like physics."

But the brain is the product of evolution—a messy, random process—and though the number sense may be lodged in a particular bit of the cerebral cortex, its circuitry seems to be intermingled with the wiring for other mental functions. A few years ago, while analyzing an experiment on number comparisons, Dehaene noticed that subjects performed better with large numbers if they held the response key in their right hand but did better with small numbers if they held the response key in their left hand. Strangely, if the subjects were made to cross their hands, the effect was reversed. The actual hand used to make the response was, it seemed, irrelevant; it was space itself that the subjects unconsciously associated with larger or smaller numbers. Dehaene hypothesizes that the neural circuitry for number and the circuitry for location overlap. He even suspects that this may be why travellers get disoriented entering Terminal 2 of Paris's Charles de Gaulle Airport, where small-numbered gates are on the right and large-numbered gates are on the left. "It's become a whole industry now to see how we associate number to space and space to number," Dehaene said. "And we're finding the association goes very, very deep in the brain."

Last winter, I saw Dehaene in the ornate setting of the Institut de France, across the Seine from the Louvre. There he accepted a prize of a quarter of a million euros from Liliane Bettencourt, whose father created the cosmetics group L'Oréal. In a salon hung with tapestries, Dehaene described his research to a small audience that included a former Prime Minister of France. New techniques of neuroimaging, he explained, promise to reveal how a thought process like calculation unfolds in the brain. This isn't just a matter of pure knowledge, he added. Since the brain's architecture determines the sort of abilities that come naturally to us, a detailed understanding of that architecture should lead to better ways of teaching children mathematics and may help close the educational gap that separates children in the West from those in several Asian countries. The fundamental problem with learning mathematics is that while the number sense may be genetic, exact calculation requires cultural tools—symbols and algorithms—that have been around for only a few thousand years and must therefore be absorbed by areas of the brain that evolved for other purposes. The process is made easier when what we are learning harmonizes with built-in circuitry. If we can't change the architecture of our brains, we can at least adapt our teaching methods to the constraints it imposes.

For nearly two decades, American educators have pushed "reform math," in which children are encouraged to explore their own ways of solving problems. Before reform math, there was the "new math," now widely thought to have been an educational disaster. (In France, it was called *les maths modernes*, and is similarly despised.) The new math was grounded in the theories of the influential Swiss psychologist Jean Piaget, who believed that children are born without any sense of number and only gradually build up the concept in a series of developmental stages. Piaget thought that children, until the age of four or five, cannot grasp the simple principle that moving objects around does not affect how many of them there are, and that there was therefore no point in trying to teach them arithmetic before the age of six or seven.

Piaget's view had become standard by the nineteen-fifties, but psychologists have since come to believe that he underrated the arithmetic competence of small children. Six-month-old babies, exposed simultaneously to images of common objects and sequences of drumbeats, consistently gaze longer at the collection of objects that matches the number of drumbeats. By now, it is generally agreed that infants come equipped with a rudimentary ability to perceive and represent number. (The same appears to be true for many kinds of animals, including salamanders, pigeons, raccoons, dolphins, parrots, and monkeys.) And if evolution has equipped us with one way of representing number, embodied in the primitive number sense, culture furnishes two more: numerals and number words. These three modes of thinking about number, Dehaene believes, correspond to distinct areas of the brain. The number sense is lodged in the parietal lobe, the part of the brain that relates to space and location; numerals are dealt with by the visual areas; and number words are processed by the language areas.

Nowhere in all this elaborate brain circuitry, alas, is there the equivalent of the chip found in a five-dollar calculator. This deficiency can make learning that terrible quartet—"Ambition, Distraction, Uglification, and Derision," as Lewis Carroll burlesqued them—a chore. It's not so bad at first. Our number sense endows us with a crude feel for addition, so that, even before schooling, children can find simple recipes for adding numbers. If asked to compute 2 + 4, for example, a child might start with the first number and then count upward by the second number: "two, three is one, four is two, five is three, six is four, *six*." But multiplication is another matter. It is an "unnatural practice," Dehaene is fond of saying, and the reason is that our brains are wired the wrong way. Neither intuition nor counting is of much use, and multiplication facts must be stored in the brain verbally, as strings of words. The list of arithmetical facts to be memorized may be short, but it is fiendishly tricky: the same numbers occur over and over, in different orders, with partial overlaps and irrelevant rhymes. (Bilinguals, it has been found, revert to the language they used in school when doing multiplication.) The human memory, unlike that of a computer, has evolved to be associative, which makes it ill-suited to arithmetic, where bits of knowledge must be kept from interfering with one another: if you're trying to retrieve the result of multiplying 7 X 6, the reflex activation of 7 + 6 and 7 X 5 can be disastrous. So multiplication is a double terror: not only is it remote from our intuitive sense of number; it has to be internalized in a form that clashes with the evolved organization of our memory. The result is that when adults multiply single-digit numbers they make mistakes ten to fifteen per cent of the time. For the hardest problems, like 7 X 8, the error rate can exceed twenty-five per cent.

Our inbuilt ineptness when it comes to more complex mathematical processes has led Dehaene to question why we insist on drilling procedures like long division into our children at all. There is, after all, an alternative: the electronic calculator. "Give a calculator to a five-year-old, and you will teach him how to make friends with numbers instead of despising them," he has written. By removing the need to spend hundreds of hours memorizing boring procedures, he says, calculators can free children to concentrate on the meaning of these procedures, which is neglected under the educational status quo. This attitude might make Dehaene sound like a natural ally of educators who advocate reform math, and a natural foe of parents who want their children's math teachers to go "back to basics." But when I asked him about reform math he wasn't especially sympathetic. "The idea that all children are different, and that they need to discover things their own way—I don't buy it at all," he said. "I believe there is one brain organization. We see it in babies, we see it in adults. Basically, with a few variations, we're all travelling on the same road." He admires the mathematics curricula of Asian countries like China and Japan, which provide children with a highly structured experience, anticipating the kind of responses they make at each stage and presenting them with challenges designed to minimize the number of errors. "That's what we're trying to get back to in France," he said. Working with his colleague Anna Wilson, Dehaene has developed a computer game called "The Number Race" to help dyscalculic children. The software is adaptive, detecting the number tasks where the child is shaky and adjusting the level of difficulty to maintain an encouraging success rate of seventy-five per cent.

Despite our shared brain organization, cultural differences in how we handle numbers persist, and they are not confined to the classroom. Evolution may have endowed us with an approximate number line, but it takes a system of symbols to make numbers precise—to "crystallize" them, in Dehaene's metaphor. The Mundurukú, an Amazon tribe that Dehaene and colleagues, notably the linguist Pierre Pica, have studied recently, have words for numbers only up to five. (Their word for five literally means "one hand.") Even these words seem to be merely approximate labels for them: a Mundurukú who is shown three objects will sometimes say there are three, sometimes four. Nevertheless, the Mundurukú have a good numerical intuition. "They know, for example, that fifty plus thirty is going to be larger than sixty," Dehaene said. "Of course, they do not know this verbally and have no way of talking about it. But when we showed them the relevant sets and transformations they immediately got it."

The Mundurukú, it seems, have developed few cultural tools to augment the inborn number sense. Interestingly, the very symbols with which we write down the counting numbers bear the trace of a similar stage. The first three Roman numerals, I, II, and III, were formed by using the symbol for one as many times as necessary; the symbol for four, IV, is not so transparent. The same principle applies to Chinese numerals: the first three consist of one, two, and three horizontal bars, but the fourth takes a different form. Even Arabic numerals follow this logic: 1 is a single vertical bar; 2 and 3 began as two and three horizontal bars tied together for ease of writing. ("That's a beautiful little fact, but I don't think it's coded in our brains any longer," Dehaene observed.)

Today, Arabic numerals are in use pretty much around the world, while the words with which we name numbers naturally differ from language to language. And, as Dehaene and others have noted, these differences are far from trivial. English is cumbersome. There are special words for the numbers from 11 to 19, and for the decades from 20 to 90. This makes counting a challenge for English-speaking children, who are prone to such errors as "twenty-eight, twenty-nine, twenty-ten, twenty-eleven." French is just as bad, with vestigial base-twenty monstrosities, like *quatre-vingt-dix-neuf* ("four twenty ten nine") for 99. Chinese, by contrast, is simplicity itself; its number syntax perfectly mirrors the base-ten form of Arabic numerals, with a minimum of terms. Consequently, the average Chinese four-year-old can count up to forty, whereas American children of the same age struggle to get to fifteen. And the advantages extend to adults. Because Chinese number words are so brief—they take less than a quarter of a second to say, on average, compared with a third of a second for English—the average Chinese speaker has a memory span of nine digits, versus seven digits for English speakers. (Speakers of the marvellously efficient Cantonese dialect, common in Hong Kong, can juggle ten digits in active memory.)

In 2005, Dehaene was elected to the chair in experimental cognitive psychology at the Collège de France, a highly prestigious institution founded by Francis I in 1530. The faculty consists of just fifty-two scholars, and Dehaene is the youngest member. In his inaugural lecture, Dehaene marvelled at the fact that mathematics is simultaneously a product of the human mind and a powerful instrument for discovering the laws by which the human mind operates. He spoke of the confrontation between new technologies like brain imaging and ancient philosophical questions concerning number, space, and time. And he pronounced himself lucky to be living in an era when advances in psychology and neuroimaging are combining to "render visible" the hitherto invisible realm of thought.

For Dehaene, numerical thought is only the beginning of this quest. Recently, he has been pondering how the philosophical problem of consciousness might be approached by the methods of empirical science. Experiments involving subliminal "number priming" show that much of what our mind does with numbers is unconscious, a finding that has led Dehaene to wonder why some mental activity crosses the threshold of awareness and some doesn't. Collaborating with a couple of colleagues, Dehaene has explored the neural basis of what is known as the "global workspace" theory of consciousness, which has elicited keen interest among philosophers. In his version of the theory, information becomes conscious when certain "workspace" neurons broadcast it to many areas of the brain at once, making it simultaneously available for, say, language, memory, perceptual categorization, action-planning, and so on. In other words, consciousness is "cerebral celebrity," as the philosopher Daniel Dennett has described it, or "fame in the brain."

In his office at NeuroSpin, Dehaene described to me how certain extremely long workspace neurons might link far-flung areas of the human brain together into a single pulsating circuit of consciousness. To show me where these areas were, he reached into a closet and pulled out an irregularly shaped baby-blue plaster object, about the size of a softball. "This is my brain!" he announced with evident pleasure. The model that he was holding had been fabricated, he explained, by a rapid-prototyping machine (a sort of three-dimensional printer) from computer data obtained from one of the many MRI scans that he has undergone. He pointed to the little furrow where the number sense was supposed to be situated, and observed that his had a somewhat uncommon shape. Curiously, the computer software had identified Dehaene's brain as an "outlier," so dissimilar are its activation patterns from the human norm. Cradling the pastel-colored lump in his hands, a model of his mind devised by his own mental efforts, Dehaene paused for a moment. Then he smiled and said, "So, I kind of like my brain." ♦

ILLUSTRATION: JOOST SWARTE