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The Time Course of Parietal Activation in Single-digit Multiplication: Evidence from Event-related Potentials

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Event-related potentials (ERPs) were used to examine the organization of brain activations during single-digit multiplication. Electrophysiological, neuropsychological, and brain-imaging data suggest that left inferior parietal areas are involved in mental calculation. We aimed at investigating the involvement of this area in simple and difficult single-digit multiplications, and at determining the time course of its activation. ERPs were recorded from 64 channels while subjects performed a sequential multiplication-verification task. Simple and difficult multiplication problems were presented either visually as arabic digits or auditorily as number words. For both modalities of input, a significant effect of difficulty was found on left and right inferior parietal electrode sites. The results suggested that simple multiplication problems may involve a short-lived activation in the left inferior parietal cortex, whereas complex problems may require longer processing which also involves the homologous right area. These findings also demonstrate the significance of ERPs as a tool for determining the temporal orchestration of brain areas involved in a cognitive task.

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Subject testing and data analysis were performed at the Center for the Cognitive Neuroscience of Attention, University of Oregon. This work greatly benefited from discussions with Laurent Cohen, Mike Posner, and Don Tucker. Paul Compton, Vonda Evans, Lynn McDougall, Eric Gorr, and Michael Murias provided crucial help during testing and data analysis. S.D. was sponsored by I.N.S.E.R.M., C.N.R.S., and the N.S.F. Equipment and subject testing were made possible by Office of Naval Research grant N-00014-89-J3013, by NIMH grants MH42129 and MH42669, and by a grant from the James S. McDonnell Foundation and Pew Memorial Trusts to support the Center for the Cognitive Neuroscience of Attention, University of Oregon.

Event-related potentials (ERPs) provide a powerful tool for determining the time course of brain activations during mental operations (Posner & Raichle, 1994; Rugg & Coles, 1995). Because electroencephalography has a temporal resolution in the range of milliseconds, it may potentially reveal the temporal sequence of brain activations that underlie cognitive processing (e.g. Dehaene, 1996; Snyder, Abdullaev, Posner, & Raichle, 1995). Although the spatial resolution of ERPs is inferior to other functional neuroimaging techniques such as positron emission tomography (PET) or functional magnetic resonance imaging (fMRI), their topography nevertheless conveys some information about the neural substrates of cognitive processes. Thus, when there is converging evidence from functional brain-imaging studies and electrophysiological experiments about the localization of the brain regions that are active during a given cognitive task, ERPs may help to delineate the time course of this activity.

In this paper, we use ERPs to study brain activity while humans calculate. During the last decade, considerable progress has been made towards understanding the mental and cerebral representations that are accessed during calculation and number processing (for a review, see Dehaene & Cohen, 1995). In particular, converging evidence has been accumulated that the left parieto-occipito-temporal region of the brain is crucial in calculation processes. One line of evidence comes from electrophysiological studies (Altenmüller, Jung, Winkler, & Landwehrmeyer, 1989; Inouye, Shinosaki, Iyama, & Mastumoto, 1993; Papanicolaou, Schmidt, Moore, & Eisenberg, 1983; Shepard & Gale, 1982). Altenmüller et al. (1989) found a preponderance of left posterior activation in 65% of his subjects as indexed by a slow negative DC-shift during a complex addition task (i.e. adding two-digit numbers). Similarly, Inouye et al. (1993) reported EEG desynchronization over the left temporo-centro-parietal areas and increased coherence between the posterior left hemisphere and frontal areas during a repeated subtraction task relative to rest. In those studies, however, sparse electrode sampling did not allow for a fine-grained localization of the possible brain electrical sources, nor was the time course of activations investigated.

Functional brain imaging studies also suggest that the inferior parietal cortex, particularly in the left hemisphere, plays a crucial role in number processing. Roland and Friberg (1985) using the Xe^{133} technique and Rueckert et al. (1996) with fMRI found increased blood flow in frontal and inferior parietal regions during mental calculation. The activations were bilateral, but with a tendency towards a greater involvement of the left hemisphere. In both experiments, the subjects had to perform a repeated subtraction task which not only requires calculation processes, but also imposes demands on working memory and on more global executive functions that may explain the observed activations in prefrontal cortex. Dehaene et al. (1996) investigated calculation and number processing in a more restricted setting. Using PET, they measured cerebral blood flow during single-digit multiplication and comparison tasks. Subjects were presented visually with two arabic digits. They had either to multiply them mentally or to select the larger digit. In both cases, they had to name the result covertly. Both multiplication and comparison activated occipital and frontal regions that could be related to visual recognition and speech output, respectively. Multiplication also activated left and right inferior parietal areas, with a non-significant tendency towards a greater activation on the left.

These findings fit with a third source of evidence, namely the data from brain-lesioned patients. Acalculia is most frequently associated with lesions in the left posterior portion of the brain (Grafman, Passafiume, Faglioni, & Boller, 1982). Within this area, the left inferior parietal cortex has been shown to be a major lesion site for calculation deficits (e.g. Benson & Weir, 1972; Benton, 1987; Gerstmann, 1940; Henschen, 1919; Warrington, 1982). Patients with left posterior lesions often have difficulty in answering single-digit arithmetic problems such as 9+2, 7×8, or 7–3. A gradient of difficulty is commonly observed in that patients often remain able to retrieve very simple multiplication problems like 2×3 while failing to carry out more difficult ones like 6×7 (e.g. McCloskey, Aliminosa, & Sokol, 1991; Warrington, 1982). This is analogous to the well-established empirical finding that, in normal subjects, larger products take more time to compute than smaller products—the so-called problem-size effect (Ashcraft, 1992). For instance, subjects are faster to compute 2×3 than 6×7). Although there have been many theories for the source of this effect (see Ashcraft, 1992, for review), one interesting possibility that has recently surfaced is that small problems may involve direct retrieval from memory whereas larger problems are solved using various strategies such as counting or decomposition (LeFevre, Sadesky, & Bisanz, 1996). Dehaene and Cohen (1995) speculate that the answers to small problems are stored as rote verbal facts and retrieved using a left cortico-subcortical circuit not involving the left inferior parietal region. Larger problems, however, have to be recoded using semantic knowledge about numbers [e.g. 6×9 would be recoded as (6×10)–6]. This semantic elaboration process would require the activation of a semantic representation of numerical quantities stored in inferior parietal areas.

The present study was designed to probe the sequence of brain activations during single-digit multiplication. In particular, we sought to measure the involvement of the left inferior parietal region in simple and difficult single-digit multiplications, and to determine the time course of its activation during calculation. For this purpose, a topographic analysis of event-related potentials was carried out using problem size as a variable of interest. We also investigated whether the left parietal activation could be observed regardless of the modality in which the multiplication problems were presented. The issue of whether mental calculation processes are modality-dependent is the focus of a longstanding debate in cognitive psychology (e.g. Campbell, 1994; Campbell & Clark, 1992; Dehaene, 1992; McCloskey, 1992). However, to the best of our knowledge, this issue has not been addressed yet using functional brain imaging.

Our experiment involved the recording of ERPs from 64 electrodes during a multiplication task. Subjects were presented sequentially with the two operands of a multiplication problem. They were left some time to compute the product mentally and were then presented with a proposed result that they had to classify as true or false. Problem size was systematically varied, as was the modality of the operands and of the proposed result (auditory or visual).

We predicted that ERPs to simple and difficult problems would begin to diverge in the interval following the presentation of the second operand, and that this divergence would be specific to electrodes located over the left inferior parietal cortex. Because difficult problems require more processing time than simple ones, they should elicit a prolonged activation on these electrodes relative to simple ones. Furthermore, since the left inferior parietal region is thought to involve an abstract representation of numerical quantities, it was expected that the topography of the problem-size effect would be similar regardless of input modality.

METHOD

Stimuli

Each multiplication problem was composed of a pair of operands with numbers ranging from 2 to 9 and a proposed multiplication result which could be either true or false. Half were simple multiplication problems with operands ranging from 2 to 5 (e.g. 2×4). The other half were difficult multiplication problems with operands ranging from 6 to 9 (e.g. 6×7). Ties were excluded from the problem set. In one half of problems, the smaller operand was presented first (e.g. 2×4), and in the other half the order of the operands was reversed (e.g. 4×2). Moreover, the disparity of the proposed multiplication result was varied. The proposed result could be either correct (e.g. 2×4=8), or false but close to the actual result (2×4=10), or false and far from the actual result (2×4=36). The proposed result always belonged to the same line or column of the multiplication table as the actual result. False results were never equal to the sum of the operands, nor did they show any phonological similarity with the actual result, nor did they include a repetition of one of the operands. However, false results always had the same parity as the actual result. According to these restrictions a total set of 72 problems was constructed (12 products × 2 digit orders × 3 disparity levels; see Appendix A). An additional set of 48 training problems was created with one operand between 2 and 5 and another between 6 and 9.

The operands and the proposed result were presented either visually as arabic digits or auditorily as number words. Half of the operand pairs were presented visually (e.g. "2" "3") and the other half were presented auditorily (e.g. /two/

/three/). Likewise, the proposed result was presented either visually ("6") or auditorily (/six/). The modality of the operands and the proposed result was varied orthogonally, resulting in four combinations (visual–visual, visual–auditory, auditory–visual, auditory–auditory). The arabic digits were presented on a standard black-and-white Macintosh monitor (white on black) equipped with a Polaroid CP-50 circular polarizing filter to reduce glare and stimulus persistence. They were displayed in a Geneva Bold 48-point font and subtended a visual angle of $1.1 \times 1.5°$ (12×16 mm) for single digit numbers and $2.2 \times 1.5°$ (26×16 mm) for two-digit numbers at a viewing distance of about 60 cm. The stimuli were presented unmasked for 150 msec, synchronous with the 15-msec refresh cycle of the monitor, within a white rectangle that remained visible throughout the whole session. The number words were presented auditorily with a loudness of about 70 dB through a loudspeaker centred below the monitor. The auditory stimuli were digitized spoken number words which were produced by a female native-English speaker. After recording, the stimuli were compressed to a duration of 150 msec for the operands and 150–350 msec for the proposed result, using a pitch-synchronous algorithm to preserve the spectral characteristics of speech. During auditory stimulation, subjects were asked to maintain fixation on the empty white frame on screen. For both visual and auditory stimuli, the two operands were presented sequentially with an interstimulus interval (ISI) of 350 msec. The ISI between the second operand and the proposed result was 1,250 msec. This relatively long ISI was used because subjects were expected to have completed the calculation by the time the proposed result was presented. The interval between the key press and the next trial was fixed at 3,000 msec (for the temporal structure of one trial, see Fig. 1).

Experimental design and procedure

Subjects were presented with a random list of multiplication problems involving two operands and a proposed result. Their task was to start computing the product immediately after the second operand had been presented, and then to compare the computed result with the proposed result. They had to decide whether the proposed result was true or false by pressing one of two response keys using the left or right hand. Responses were measured via two microswitches, 2 cm apart, connected to the millisecond timer of a National Instrument NB-DMA-8 card. Instructions emphasized both speed and accuracy.

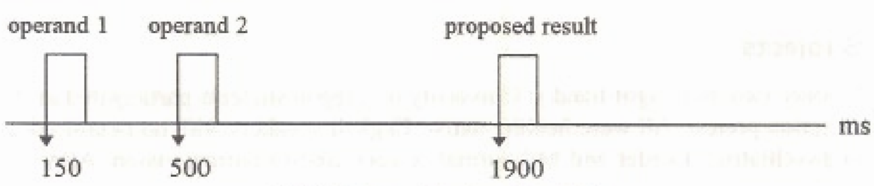


FIG. 1. Temporal structure of one trial.

Subjects first participated in a pretest without ERP recording to familiarize them with the task and to reject subjects with exceptionally high error rates or with very slow or fast performance. The pretest comprised four consecutive blocks. Within each block, one of the four possible combinations of operand modality and result modality was presented (visual–visual, visual–auditory, auditory–visual, auditory–auditory). Each block included 36 problems which were drawn randomly from the total set such that only one operand order for each product—either "smaller digit first" (e.g. 2×4) or "larger digit first" (e.g. 4×2)—was realized. Thus, in the pretest only one half of the problems were presented. The first block was preceded by 10 training trials and the remaining three blocks by 6 training trials. The presentation of the blocks was varied in a counterbalanced fashion. The assignment of "true" and "false" responses to the left and the right hands was counterbalanced across subjects. After each block, the performance of each subject was examined by the experimenter and the subject was asked to be faster if the mean RT was below 700 msec and more accurate if the mean error rate exceeded 10%. Only subjects who met these criteria after the fourth block were included in the main experiment. The results of the pretest were not analysed further because the responses were affected differentially by experimenter feedback.

On a subsequent day, the selected subjects came for the main experiment in which ERPs were recorded. Subjects were now presented with two main blocks of trials, each being divided into four short blocks with a given combination of operand and result modality. Thus, as in the pretest, the four combinations of operand and result modality were presented separately. In one main block, the "true" response was assigned to the right-hand key and the "false" response to the left-hand key. This assignment was reversed in the second main block. Subjects started the first main block with the hand assignment opposite to that used in the pretest. The total set of 72 problems was presented four times, once for each combination of operand and result modality. One half of the problems were presented in the first main block and the other half in the second one by assigning the operand order of a given product randomly to the main blocks. It was controlled in such a way, however, that each operand order was realized. As in the pretest, each short block consisted of 36 experimental trials. There were also 10 training trials at the beginning of the first block and 6 for the remaining ones. Unlike in the pretest, no feedback was given to the subjects until the end of the session.

Subjects

Twenty-two male right-handed University of Oregon students participated in the selection pretest. All were healthy native English speakers with no neurological or psychiatric disorder and had normal or corrected-to-normal vision. After the pretest, 4 subjects were rejected for not meeting the inclusion criteria, and 2 additional subjects refused to continue the experiment. In the ERP test, 4 subjects had to be excluded from analysis, one because of programme error and 3 because of less than 66% artifact-free trials. The remaining 12 subjects had an average age of 21.9 years (range 18–29).

ERP Recording

Scalp voltages were collected using a 64-channel Geodesic Sensor Net (Tucker, 1993). The net consists of 58 Ag/AgCl electrodes encased in sponges soaked in KCl electrolyte solution. The electrodes occupy the vertices of a triangular mesh of elastic line, which ensures their regular repartition on the surface of the scalp with a spacing of about 4 cm. The net was applied in anatomical reference to the canthomeatal line and was supplemented with 8 independent electrodes at mastoid, nasion, inion, infraorbital and external canthus locations. The inion electrode was connected to the ground, and the right mastoid electrode was used as reference. Electrical signals were amplified by a 64-channel custom-built AC-coupled high-impedance amplifier with a band-pass of 0.1–50 Hz (3 dB/octave attenuation) and a 60-Hz notch filter. Scalp voltages were digitized by a Macintosh II computer using a National Instruments muliplexor card AMUX-64, 16-bit analog-to-digital converter card NB-MIO-16, and direct memory access card NB-DMA-8 and stored on the hard disk for off-line analysis. Data were collected for 386 samples at a sampling rate of 125 Hz starting 150 msec before the onset of the first operand. The recording epoch covered a total of 3,000 msec.

Artifact Rejection, Signal Extraction and Graphical Rendering

Trials were edited automatically, and all trials contaminated with ocular or movement artifacts were discarded. Moreover, channels were considered as bad and were excluded for a given trial if their voltage exceeded 100 μV or if there was a sharp variation in voltage of more than 80 μV in consecutive samples. Trials with correct responses were averaged synchronous to stimulus onset. The averaged ERPs were digitally band-pass filtered with an upper cut-off at 30 Hz and a lower cut-off at 0.15 Hz using a two-pole double Butterworth filter. After filtering, voltage data were aligned to the 150-msec pre-stimulus baseline. The average-reference transform was applied to obtain a reference-independent estimation of scalp voltage (Bertrand, Perrin, & Pernier, 1985; Curran, Tucker, Kutas, & Posner, 1993; Tucker, Liotti, Potts, Russell, & Posner, 1994), and the reconstructed right mastoid reference was added to the data resulting in a total of 65 data channels. The average-reference voltage data were interpolated across the surface of the scalp using spherical splines (Perrin, Pernier, Bertrand, & Echallier, 1989). The reconstructed voltage surface could then be visualized using a grey scale for coding the different levels of voltage on the head.

Statistical Analysis

Two strategies were used for statistical analysis. First, a nonparametric Wilcoxon matched pairs signed-rank test was applied to each time sample and each electrode site to reveal consistent differences among two conditions (e.g. simple vs. difficult problems, visual vs. auditory operands, etc.). An effect was considered to be significant if reliable differences at the .05 level were found for at least five consecutive samples (40 msec). The onset of an effect was then determined as the first sample with a significant divergence.

Second, bilateral pairs of electrodes were selected for parametric analysis of specific time windows. Electrode pairs were selected a priori according to the theoretical predictions (e.g. inferior parietal electrodes) or according to the maxima of the main ERP-components observed in the experiment. The time windows for analysing the interval between the offset of the second operand and the onset of the proposed result (i.e. the interval during which the multiplication result had to be computed) were determined according to the time course of the two major ERP deflections observed in this part of the recording epoch (positive slow wave, negative slow wave).[1] The time intervals covering each of these deflections were split into two smaller epochs resulting in a total of four time windows (rise and fall of positive slow wave, first half and second half of negative slow wave). The following time windows (with respect to the onset of the second operand) were obtained: 270–397 msec post-stimulus (rise of positive slow wave); 398–629 msec post-stimulus (fall of positive slow wave); 630–1,013 msec post-stimulus (first half of negative slow wave); 1,014–1,399 msec post-stimulus (second half of negative slow wave). The following pairs of electrodes were selected for analysis in specific time windows: P3A/P4A (inferior parietal), T5A/T6A (superior temporal), C3P/C4P (centro-parietal), C3SP/C4SP (superior centro-parietal), Fp1S/Fp2S (prefrontal), F3IP/F4IP (inferior frontal), F3S/F4S (superior frontal).

[1]Effects of stimulus encoding in the interval between the first and the second operand were also observed, but they are reported here only briefly because the experiment is mainly focused on arithmetic fact retrieval. The main result was that number magnitude affected visual N1 and auditory P2 amplitude. Smaller numbers (i.e. operands of simple problems or the smaller operand of one operand pair) elicited larger amplitudes in both the visual and auditory modality than larger numbers (i.e. operands of complex problems or the larger operand of one operand pair). Similarly, peak latencies were slightly delayed for larger numbers compared to simple numbers. Probably, it took longer to encode larger operands than simple ones. A plausible explanation for this effect of number magnitude on these sensory ERPs is that smaller and larger numbers differ in word frequency. In all languages, the frequency of numerals is inversely related to numerical size (Dehaene & Mehler, 1992). Word frequency may therefore affect the stage of stimulus identification in which the visual or auditory lexicon is accessed.

Separate ANOVAs were performed for visual and auditory operands on the average voltage of each time window with experimental condition (problem size, operand order or result modality) and hemisphere as factors. The number of trials did not allow for an analysis of interactions between the experimental factors. When appropriate, degrees of freedom were adjusted according to the method of Greenhouse-Geisser, and only corrected significance levels are reported. The level of significance testing was $p = .05$.

RESULTS

Electrophysiological Data

Problem-size Effect

After the presentation of the second operand, a posterior positivity emerged which was accompanied by a frontal negativity. The posterior positivity was maximal on centro-parietal electrodes with a greater amplitude on left than on right parietal sites as it can be seen from Figs. 2A and 2B. For visual operands, the ERPs to simple and difficult problems started to diverge at 334 msec on left temporo-parietal electrodes (T5A, T5P, P3A). Simple problems elicited a greater positivity than difficult ones. The parietal positivity peaked at 390 msec after the second operand for simple problems and at 414 msec for difficult ones. After reaching the peak, the waves converged at 438 msec. The ERPs started to diverge again at 670 msec bilaterally on centro-parietal electrodes (P3A/P4A, P3S/P4S, C3SP/C4SP) with a greater divergence on left-sided electrodes. In this part of the recording epoch, a slow negative ramping wave was quickly developing for simple problems, whereas for difficult problems the positivity faded only slowly. This divergence was maintained until about 1,150 msec after the second operand (see also Fig. 3 for the grand-averages of the described ERP waves). The topography of the early and later problem-size effects are shown in Figs. 2B and 2C, respectively, as a subtraction of ERPs to complex and simple problems.

A similar result pattern was also obtained for auditory operands. The left parietal positivity peaked at 398 msec after the second operand for simple problems and at 470 msec for difficult ones. The waves to simple and difficult problems started to diverge at 294 msec on left temporo-parietal electrodes (T5P, P3A, P3S) and converged again at 440 msec. Like in the visual condition, voltage to simple problems was more positive than to difficult ones and the positivity to simple problems also faded more rapidly, developing a ramping negative wave. ERPs to difficult problems remained more positive than to simple ones, resulting in a divergence of the waves starting at 742 msec and extending until 1,286 msec on inferior and centro-parietal electrodes (P3A, C3P, C3SP). As in the visual condition, the divergence tended to be larger in the left hemisphere. As an inspection of the voltage maps in Figs. 2B and 2C

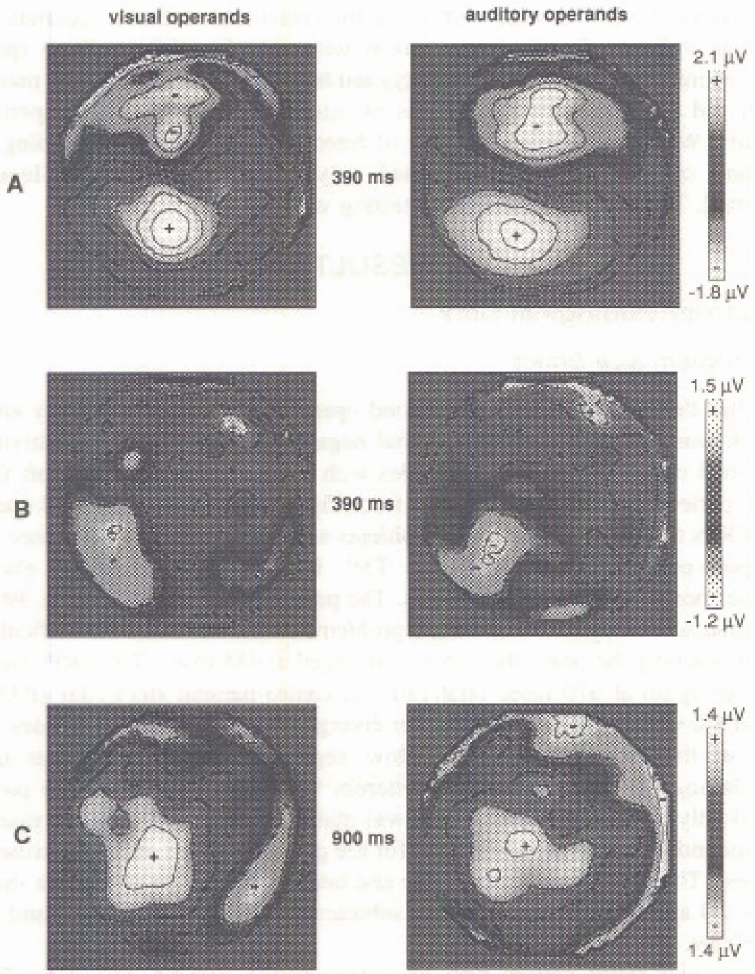


FIG. 2. Two-dimensional voltage maps from average-referenced grand average ERPs at different times in the epoch after second operand presentation (times are given with respect to the onset of the second operand). Each image shows a view of the top of the head with the anterior sites up and the posterior sites down. Scalp voltage is coded using the linear grey scale shown (scaling varies for each set). Positive and negative polarity of voltage is indicated by a plus (+) and minus sign (-), respectively. (A): Topography of ERPs to visual and auditory operands at 390 msec showing the left centro-parietal scalp distribution of the posterior positive slow wave. (B): Topography of the voltage difference between complex and simple multiplication problems at 390 msec. Note the similar left temporo-parietal scalp distribution of the problem-size effect for visual and arabic operands. (C): Topography of the voltage difference between complex and simple multiplication problems at 900 msec. Note the inversed polarity of the parietal problem-size effect ERPs to complex problems were, in this part, more positive than to simple problems.

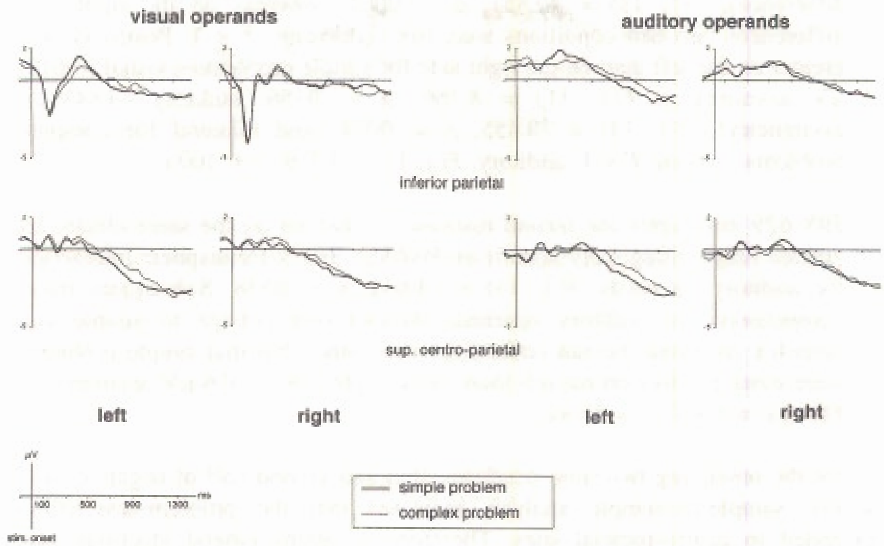


FIG. 3. Selected ERP waveforms of average-referenced grand-averaged data as a function of problem size on inferior parietal (P3A/P4A) and superior centro-parietal electrode sites (C3SP/C4SP). The onset of the second operand is indicated by the *y*-axis. The graph shows the early (from about 300 msec to 450 msec) and the late (from about 700 msec to 1,200 msec) problem-size effect.

demonstrates, the topography of the problem-size effect was quite similar for visual and auditory operands. The time course of activations was also comparable in both conditions. The effect started for both conditions with a clear left-lateralized topography and extended later on in the same region, but now involving both the left and right hemispheres. However, the divergence started earlier and was maintained longer in the auditory condition than in the visual condition.

The findings from sample-by-sample analysis were confirmed by repeated measurement ANOVAs on mean voltage of prespecified time windows for the left and right inferior parietal electrode (P3A/P4A). (For the rationale of time window and electrode selection, see the method section.)

270–397 msec after the second operand: A Problem Size × Hemisphere interaction was obtained for both visual, $F(1, 11) = 5.516$, $p = .0386$, and auditory operands, $F(1, 11) = 21.776$, $p = .0007$. This interaction indicates that problem size affected the ERPs on the left side, but not on the right side. Mean comparisons revealed that on the left inferior parietal electrode voltage to simple problems was more positive than to complex problems—visual (0.602-μV difference), $F(1, 11) = 7.229$, $p = .0211$; auditory (0.646-μV difference), $F(1, 11) = 40.851$, $p = .0001$—whereas on the right side differences between conditions were not significant, $F < 1$. Positivity was greater on the left than on the right side for simple problems—visual (0.640-µV asymmetry), $F(1, 11) = 8.166$, $p = .0156$; auditory (0.848-µV asymmetry), $F(1, 11) = 70.455$, $p = .0001$—and bilateral for complex problems—visual, $F < 1$; auditory, $F(1, 11) = 3.219$, $p = .1003$.

398–629 msec after the second operand. Analysis on the same electrodes yielded only a marginally significant Problem Size × Hemisphere interaction for auditory operands, $F(1, 11) = 4.662$, $p = .0538$. Subsequent mean comparisons for auditory operands showed that voltage to simple and complex problems did not differ from each other, but that simple problems were more positive on the left than on the right side (0.416-µV asymmetry), $F(1, 11) = 8.467$, $p = .0142$.

For the remaining two time windows (first and second half of negative slow wave), sample-by-sample analysis indicated that the problem-size effect extended to centro-parietal sites. Therefore, a centro-parietal electrode pair (C3SP/C4SP) was included in parametrical analysis in separate ANOVAs.

630–1,013 msec after the second operand: On P3A/P4A, the problem-size effect was significant in the auditory condition (0.300-µV difference), $F(1, 11) = 6.822$, $p = .0242$, whereas in the visual condition only a non-significant tendency was obtained (0.200-µV difference), $F(1, 11) = 2.659$, $p = .1312$. On C3SP/C4SP, the main effect of problem size reached significance for both visual and auditory operands: visual (0.518-µV difference), $F(1, 11) = 12.271$, $p = .0049$; auditory (0.406-µV difference), $F(1, 11) = 5.644$, $p = .0368$. Complex problems elicited a greater positivity than simple problems. For auditory operands this difference tended to be greater in the left than in the right hemisphere: P3A/P4A, $F(1, 11) = 2.996$, $p = .1114$; C3SP/C4SP, $F(1, 11) = 2.839$, $p = .1202$.

1,014–1,399 msec after the second operand: For auditory operands a marginally significant Problem Size × Hemisphere interaction was observed on C3SP/C4SP, $F(1, 11) = 4.122$, $p = .0672$, due to a left lateralized problem-size effect. For visual operands, however, problem size did not reach significance.

Effect of operand order

Sample-by-sample analysis revealed that in the auditory condition the waves to "smaller operand first" and to "larger operand first" started to diverge at about 290 msec after the second operand on superior temporal and prefrontal

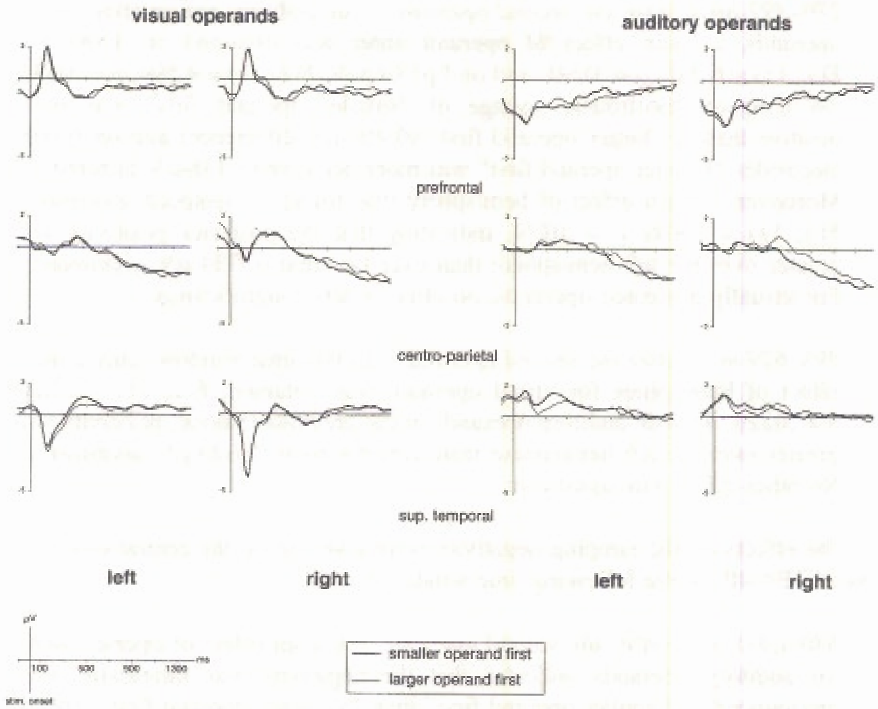


FIG. 4. Selected ERP waveforms of average-referenced grand-averaged data as a function of operand order on prefrontal (Fp1S/Fp2S), centro-parietal (C3P/C4P), and superior temporal electrode sites (T5A/T6A). The onset of the second operand is indicated by the *y*-axis. The graph shows the effect of operand order, which was only prominent for auditory operands.

electrodes (T5A/T6A, F3IP/F4IP, Fp1S/Fp2S, Fpz). On temporal electrodes, the ERP to "smaller operand first" was more positive than to "larger operand first", whereas the reversed pattern was observed on frontal electrodes. The waves converged again at 370 msec. Later in the epoch, from 860 msec to 1,206 msec a difference between the two conditions was observed on bilateral pairs of electrodes close to the midline (C3P/C4P, C3SP/C4SP, F3S/F4S). In this interval, a ramping negativity developed more strongly in the "smaller operand first" condition than in the "larger operand first" condition. For visually presented operands, no significant effect of operand order was found in sample-by-sample analysis (see also Fig. 4).

For parametrical analysis of the early fronto-temporal operand-order effect, prefrontal (Fp1S/Fp2S) and temporal pairs of electrodes (T5A/T6A) were selected and separate ANOVAs were carried out on mean voltage of the following time windows:

270–397 msec after the second operand: For auditory presentation of the operands, a main effect of operand order was obtained on T5A/T6A, $F(1, 11) = 6.286$, $p = .0291$, and on Fp1S/Fp2S, $F(1, 11) = 4.594$, $p = .0553$. On temporal electrodes, voltage of "smaller operand first" was more positive than to "larger operand first" (0.466-µV difference) and on frontal electrodes "smaller operand first" was more negative (0.438-µV difference). Moreover, a main effect of hemisphere was found on temporal electrodes, $F(1, 11) = 7.826$, $p = .0174$, indicating that the posterior positivity was greater over the left hemisphere than over the right (0.734-µV asymmetry). For visually presented operands, no effect reached significance.

398–629 msec after the second operand: In this time window, only a main effect of hemisphere for visual operands was obtained, $F(1, 11) = 7.016$, $p = .0226$. As with auditory operands in the previous interval, positivity was greater over the left hemisphere than over the right (0.620-µV asymmetry). No other effect was significant.

The effects on the ramping negativity were assessed on the central electrode pair C3P/C4P for the following time windows:

630–l,013 msec after the second operand: A main effect of operand order for auditory operands indicates that the negativity was bilaterally more pronounced to "smaller operand first" than to "larger operand first" (0.624-µV difference), $F(1, 11) = 8.614$, $p = .0136$. In the visual condition, no effect reached significance.

1,014–1,399 msec after the second operand: In the auditory condition the main effect of operand order was still significant, $F(1, 11) = 6.224$, $p = .0298$. Voltage to "smaller operand first" was again more negative than to "larger operand first" (0.304-µV difference). No other significant effect was found in this interval.

Effect of result modality

The expected modality of the proposed result affected ERPs in the epoch following the presentation of the second operand, starting at 250 msec for both auditory and visual operands. The effect was found on inferior parietal and on adjacent temporal electrodes (P3A/P4A, T5P/T6P). The topography of this effect, however, was different for visual and auditory operands: For visual operands it was left lateralized, whereas for auditory operands it was right lateralized and extended to more inferior posterior electrodes (T6I, O2LI). Starting at 540 msec for visual and at 750 msec for auditory operands, a left lateralized effect on inferior frontal electrodes was observed (particularly on

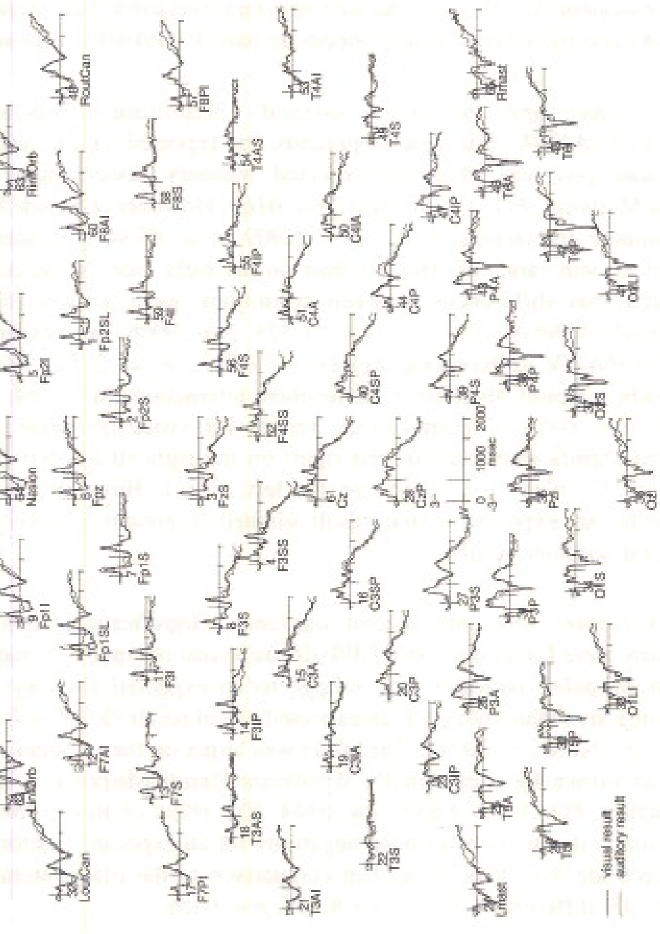
F3IP, an electrode that is located approximately over Broca's area) which continued until about 850 msec. This effect appears as a greater negativity for an expected auditory result than for a visual result. Just shortly before the proposed result was presented, the waves diverged bilaterally on superior frontal sites (F3S/F4S; at about 1,100 msec for auditory and 1,260 msec for visual operands) with a visual result more negative than an auditory result (see also Fig. 5).

For subsequent ANOVAs of the first two time windows, the inferior parietal (P3A/P4A) and the inferior frontal electrode pair (F3IP/F4IP) were selected.

270–397 msec after the second operand: Significant effects were found only on P3A/P4A. For visual operands, an expected visual result elicited a greater positivity than an expected auditory result: main effect of Result Modality, $F(1, 11) = 8.021$, $p = .0163$. However, a Result Modality × Hemisphere interaction, $F(1, 11) = 5.909$, $p = .0334$, indicates that this difference was larger on the left than on the right side. Mean comparisons revealed that differences between conditions were reliable on the left (0.654-μV difference), $F(1, 11) = 21.023$, $p = .0008$, but not on the right side (0.164-μV difference), $F(1,11) = 1.317$, $p = .2755$. For auditory operands, a Result Modality × Hemisphere interaction was found, $F(1, 11) = 5.299$, $p = .0419$. Contrary to the results for visual operands, conditions differed significantly only on the right: on the right (0.348-μV difference), $F(1, 11) = 7.620$, $p = .0185$; on the left, $F < 1$. However, as for visual operands, an expected visual result elicited a greater positivity than an expected auditory result.

398–629 msec after the second operand: Significant effects of result modality were found only on F3IP/F4IP for visual operands. A main effect of Result Modality indicates that voltage to an expected auditory result was more negative than voltage to an expected visual result (0.356-μV difference) $F(1, 11) = 6.804$, $p = .0243$. The effect was larger on the left than on the right side, as shown by a marginally significant Result Modality × Hemisphere interaction, $F(1, 11) = 4.632$, $p = .0544$. The effect of Result Modality was particularly due to a pronounced negativity for an expected auditory result on the electrode over Broca's area in comparison to the contralateral electrode (0.402-μV difference), $F(1, 11) = 8.169$, $p = .0156$.

For the last two time windows, which cover the interval of the ramping negativity, the effect of result modality was assessed again on the inferior frontal electrode pair (F3IP/F4IP) and additionally on the superior frontal electrode pair (F3S/F4S) where the result modality related negativity was most prominent:



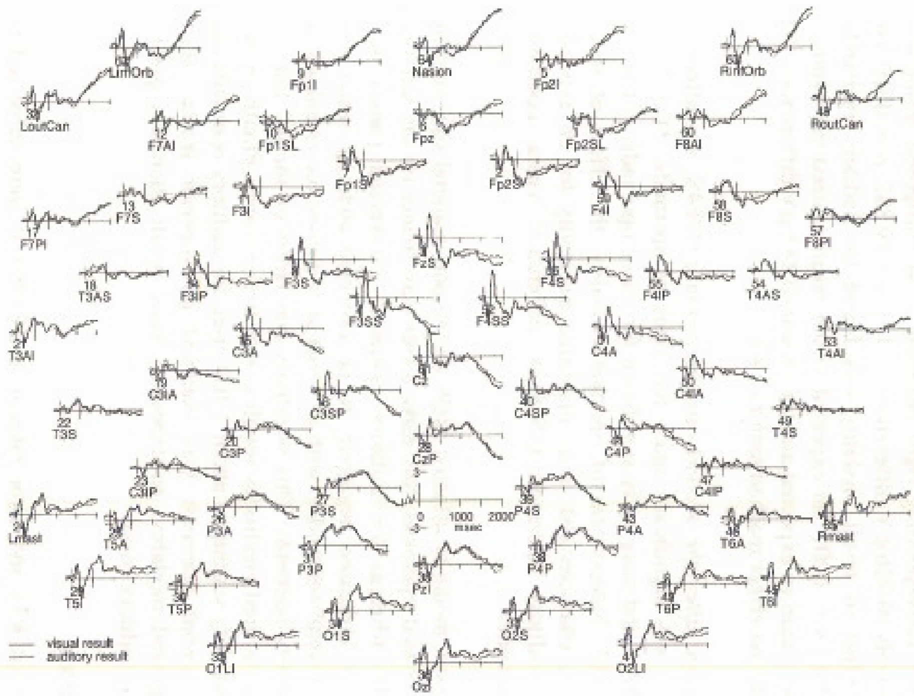


FIG. 5. Average-referenced grand-averaged ERPs to visual operands (top) and to auditory operands (bottom) as a function of result modality (visual result = thick curve, auditory result = thin curve). Each graph shows a 2,050-msec epoch, including a 150-msec baseline prior to the onset of the first operand and ending before the onset of the proposed result, at a given electrode site (top = anterior, bottom = posterior, left = left-hemisphere sites, right = right-hemisphere sites). The onset of the operands is indicated by vertical lines. Note the left inferior frontal effect of result modality with an onset at about 540 msec for visual operands and at about 750 msec for auditory operands and the later superior frontal 150
effect just before the presentation of the proposed result.

630–1013 msec after the second operand: For visual operands, the effect of Result Modality was still significant on F3IP/F4IP (0.384-μV difference), $F(1, 11) = 10.010$, $p = .0090$; however, there was now only a tendency for a left-lateralized effect (Result Modality × Hemisphere), $F(1, 11) = 3.235$, $p = .0995$. For auditory operands a significant Result Modality × Hemisphere interaction was obtained, $F(1, 11) = 5.230$, $p = .0430$. Mean comparisons revealed a greater negativity on the left inferior frontal electrode (F3IP) compared to the corresponding right electrode if the proposed result was presented auditorily (0.726-μV difference), $F(1, 11) = 19.882$, $p = .0010$. On F3S/F4S, an effect of result modality was found for auditory operands, $F(1, 11) = 7.366$, $p = .0201$: an expected visual result elicited bilaterally a greater negativity than an expected auditory result (0.282-μV difference). For visual operands, no effect reached significance.

1,014–1,399 msec after the second operand: On F3S/F4S, the effect of result modality was significant again for auditory operands, $F(1, 11) = 11.922$, $p = .0054$, and marginally significant for visual operands, $F(1, 11) = 4.454$, $p = .0585$. Independent of operand modality, the effect of result modality can be characterized as an increased negativity for an expected visual result (auditory operands: 0.548-μV difference; visual operands: 0.426-μV difference).

In summary, result modality affected ERPs on temporo-parietal sites starting at 250 msec. For visual operands, this early effect was greater on the left side than on the right, whereas for auditory operands a reversed pattern was observed. Later, an increased negativity to an expected auditory result was found on inferior frontal electrodes, with an onset at 540 msec for visual and at 750 msec for auditory operands. This negativity was larger on the left than on the right side and covered electrodes roughly over Broca's area. Finally, a late effect of result modality starting at about 1,100 msec for auditory operands and 1,260 msec for visual operands was obtained on superior frontal sites. Regardless of operand modality, an expected visual result elicited a greater negativity than an auditory result.

Behavioural data

Both median correct RTs and error rates of the ERP test were subjected to repeated measurement ANOVAs with operand modality, result modality, problem size, operand order, and result disparity as factors. Since the behavioural data did not directly reflect the calculation process itself, but rather a composition of calculation and result verification, the results of this analysis are reported only briefly.

Overall median correct RT was 412 msec. The ANOVA on reaction time yielded main effects of problem size, $F(1, 11) = 10.83$, $p = .0072$, and result disparity, $F(2, 22) = 12.51$, $p = .0002$. Verifications of complex problems were 30 msec slower than verifications of simple problems. The main effect of result disparity indicated that accepting a correct proposed result was faster than rejecting a false result The effect of the numerical distance of false results from the correct result was not significant, $F(1, 11) = 3.40$, $p = .0921$. However, a Problem Size × Result Disparity interaction, $F(2, 22) = 4.89$, $p = .0174$, was also found. A subsequent contrast analysis revealed that this interaction was mainly due to the presence of a distance effect for complex problems, but not for simple ones. For simple problems, close and far false proposed results were rejected equally fast, whereas for complex problems rejecting a far false result was faster than rejecting a close false result. Finally, disparity interacted with result modality, $F(2, 22) = 4.12$, $p = .0302$. Rejecting a visual result was faster than rejecting an auditory result, whereas when accepting a correct result no effect of result modality was found.

For error rates, the following picture was obtained. Difficult problems elicited more errors (5%) than simple problems (2.3%), $F(1, 11) = 10.83$, $p = .0072$. Thus, the problem-size effect was reflected not only in reaction times, but also in error rates. For "true" responses (7%) the error rates were higher than for "false" responses (close: 2.2%; far: 1.8%), which was manifested in a main effect of disparity, $F(1,11) = 13.02$, $p = .0002$. The high error rate for "true" responses was probably due to a speed–accuracy trade-off, because the "true" responses were also faster than "false" responses. Furthermore, the error rate for "true" responses was higher for auditory operands (9.0%) than for visual operands (5.0%: Operand Modality × Disparity), $F(2, 22) = 3.82$, $p = .0376$. Finally, interaction effects involving result modality indicated that there was a particular difficulty associated with visual results. When the proposed result was presented visually, error rate increased when the problem was complex (Problem Size × Result Modality), $F(1, 11) = 10.16$, $p = .0086$, when the operands were presented auditorily (Operand Modality × Result Modality), $F(1, 11) = 7.74$, $p = .0179$, when the larger operand of one operand pair was presented first (Operand Order × Result Modality), $F(1, 11) = 10.29$, $p = .0083$, and particularly when a "true" response was required (Operand Order × Disparity × Result Modality), $F(2, 22) = 8.41$, $p = .0019$.

DISCUSSION

The present study was carried out to investigate the time course of the inferior parietal activation during single-digit multiplication. It was expected that ERPs to simple and complex problems would diverge on left inferior parietal electrodes and that complex problems would elicit a prolonged activity in this area. Furthermore, we expected the left inferior parietal area to be involved in calculation regardless of the input modality—whether the problem was presented visually using arabic digits or auditorily using number words. These predictions were supported by the data. We found a left temporo-parietal effect of problem size starting at about 300 msec after operand presentation. Simple problems elicited a rapidly increasing positivity in this area; the latter rose more slowly for complex problems. The scalp topography of this effect was remarkably similar for visual and auditory operands, suggesting that the left inferior parietal area was active during mathematical fact retrieval regardless of the input format. About 700 msec following the second operand, differences in ERPs to simple and complex problems were also obtained over left and right centro-parietal areas, with a weak tendency for a greater activation on the left. At this point, complex problems showed a sustained positivity, whereas simple problems were developing a negativity. Thus, the polarity of the problem-size effect had changed in this time window, with ERPs to complex problems now being more positive than ERPs to simple problems. Again, the topography of the problem-size effect was similar for both input modalities.

As discussed in more detail below, we interpret these results as showing that left inferior parietal cortex is active starting about 300 msec following presentation of the operands of a multiplication problem. If the sustained positivity to complex problems is a valid index of ongoing task processing, the findings also suggest that processing of simple problems is completed by about 600 msec, whereas it seems to be prolonged for complex problems.

Apart from the predicted problem-size effect, other unexpected effects were also found: (i) an operand-order effect specific for auditory operands starting at 290 msec, (ii) an effect of result modality on parietal electrodes starting at about 250 msec and on left inferior frontal electrodes at about 540 msec for visual and at 750 msec for auditory operands. These earlier effects can be distinguished from later expectancy effects starting at about 1,100 msec over superior frontal areas.

In this discussion, we first examine the problem-size effect in greater detail before moving on to the operand-order effect and the effect of the modality of the proposed result.

Problem-size Effect

The topography and the time course of the parietal problem-size effect confirm and extend previous findings concerning the brain areas involved in mental calculation. The early left temporo-parietal effect fits with data from brain-lesioned patients which demonstrate that the left inferior parietal cortex is a major lesion site for calculation deficits (Benson & Weir, 1972; Benton, 1987; Gerstmann, 1940; Henschen, 1919; Warrington, 1982). The later bilateral parietal effect of problem size suggests that with increased processing time both hemispheres become engaged in the calculation process. Hence, it seems that computing the result of complex problems may require the involvement of both the left and the right parietal cortex, whereas simple problems are rapidly processed by the left parietal cortex. This interpretation is supported by the fact that, in the interval from 270 to 397 msec, ERPs were left lateralized for simple problems, whereas for complex problems ERPs were bilateral with only a tendency towards left lateralization.

Further evidence for a stronger involvement of right parietal areas with increased arithmetic-task difficulty stems from an EEG study by Earle (1985) using EEG-desynchronization (i.e. decreased alpha power) as an index of activation. Earle (1985) found a relatively stronger left compared to right parietal activation for simple to moderate tasks, whereas alpha power asymmetry decreased for the most difficult task. Hence, difficult arithmetic tasks seem to involve both left and right parietal areas. The findings of the present experiment are also generally in accord with the results of several functional brain-imaging studies which found a bilateral inferior parietal activation during calculation, with only a tendency for a greater activation on the left (Dehaene et al., 1996; Roland & Friberg, 1985; Rueckert et al., 1996). Additional evidence for the significance of parietal cortex in arithmetic tasks is provided by Abdullaev and Melnichuk (1996) who recorded neuronal activity from depth electrodes. They found a significant increase of neuronal firing rate in both left and right parietal cortices during counting and arithmetic operations (i.e. adding and subtracting two- and single-digit numbers), but not during reading the same numbers. Neuronal activity started at about 400 msec and lasted until 800 msec after stimulus onset. However, problem size was not varied systematically in these studies, so that the contribution of task difficulty to the left and right parietal activations could not be assessed.

Difficulty related modulation of ERP waves has been observed in a variety of tasks (for a review see Bierbaumer, Elbert, Canavan, & Rockstroh, 1990; Rösler & Heil, 1991; Ruchkin, Johnson, Mahaffey, & Sutton, 1988). However, scalp topography of these difficulty-related ERP effects varied with the specific task, suggesting that different neural generators are involved. Ruchkin et al. (1988), for example, observed a parietal slow wave which was related to perceptual difficulty. Rösler, Schuhmacher, and Sojka (1990) varied task demands using a mental rotation and a mental arithmetic task. The arithmetic task included adding, subtracting, or multiplying two-digit and single-digit numbers. In their study, mental arithmetic was associated with a slow negative wave over the left parietal scalp and mental rotation with a slow negativity over right parietal areas. The different arithmetic operation produced frontal slow-wave effects that were replicated by Rösler and Heil (1991). Possibly these frontal effects reflect different demands on working memory and executive function imposed by the different arithmetic operations (Dehaene & Cohen, 1995). Dehaene (1996) conducted an ERP study on number comparison and manipulated the difficulty of comparison by systematically varying the numerical distance between the numbers to be compared. In this study a bilateral effect was found on electrodes situated near the parieto-occipito-temporal junction of both hemispheres as in the present study, but with a larger effect on the right. The topography of the left inferior parietal effect found in the present study is clearly distinct from the effects observed in the studies mentioned above. This renders it improbable that the effect only reflects generic task difficulty. The parietal problem-size effect is most likely specifically related to the multiplication task.

From our results, the following temporal sequence of parietal activations can be suggested. Arithmetic fact retrieval initially engages the left inferior parietal cortex and involves at a later time (at about 600 msec) bilateral parietal areas (with only a tendency for a greater left-side activation). Simple problems seem to be processed more rapidly, as indicated by a relatively fast rise and fall of the parietal positivity. Complex problems exhibit a slower rise and fall. The extended positivity to complex problems may reflect prolonged task processing in parietal areas. The suggested time course of parietal activity is remarkably compatible with Abdullaev and Melnichuk (in press), who found an increase of neuronal firing rate during arithmetic operations in the time interval from 400 msec to 800 msec. This interpretation is also in line with findings of Pauli et al. (1994), who observed a delayed offset of a parietal positive slow wave for complex compared to simple problems. Thus, the behavioural problem-size effect, which is manifested in increased RTs and error rates (Ashcraft, 1992; Campbell, 1994), is reflected neurophysiologically by a prolonged parietal activation. An additional, non-exclusive explanation for the different time course of ERPs to simple and complex problems may also be proposed in terms of an increased latency jitter for ERPs to complex problems. Increased latency jitter in single trial ERPs would cause a smeared and extended average ERP wave with attenuated amplitude. Still, the results would suggest that complex problems are processed more slowly, therefore eliciting waves with delayed (and probably more variable) peak latencies.

What is the possible role of the inferior parietal region in number processing? Grafman (1988) has suggested that the left inferior parietal region holds a memory of arithmetical facts. This interpretation is certainly not ruled out by the present data. The parietal problem-size effect would then simply reflect the arithmetic fact-retrieval process per se. Arithmetic fact retrieval for simple problems would be performed faster than for complex problems since subjects are more trained in solving simple problems. As a consequence, the left parietal cortex would be active only for a short period of time for simple problems and would exhibit a sustained activation for complex problems. The only difficulties for this view of inferior parietal function is that it fails to account for other neuropsychological data. For instance, it does not readily account for the cases of arithmetic fact-retrieval impairments in patients with left frontal or basal ganglia lesions whose temporo-parietal cortex is intact (e.g. Lucchelli & De Renzi, 1993; Hittmair-Delazer, Semenza, & Denes, 1994). It also fails to explain why parietal areas are also active during number-processing tasks outside of calculation per se, such as number comparison, which do not involve arithmetic fact retrieval, but access to a number magnitude representation (Dehaene, 1996; Dehaene et al., 1996).

Based on a review of such neuropsychological and brain-imaging findings, Dehaene and Cohen (1995) have proposed an alternative to the view that inferior parietal cortex is the memory store for arithmetic facts. They suggest that the left inferior parietal gyrus and its homologue in the right hemisphere hold a representation of the quantitative or magnitude features of numbers. This view is supported by the above-mentioned ERP study on number comparison conducted by Dehaene (1996). Number comparison is a task that presumably involves automatic access of magnitude representations (Dehaene, 1992; Dehaene & Akhavein, 1995). As already outlined, numerical distance between the numbers to be compared elicited a bilateral inferior parietal effect that was larger on the right. Most importantly, the topography and the latency of the distance effect were quite similar whether the numbers were presented in arabic or verbal notation. Hence, this study provides evidence for a modality-independent magnitude representation tentatively localized to the left and right inferior parietal regions.

According to Dehaene (1992) and Dehaene and Cohen (1995), simple arithmetic facts such as $2\times3=6$ are stored and retrieved from a rote verbal memory without regard to the quantities involved. Even for simple multiplications or additions, however, memory is not complete or error-free (e.g. Campbell & Graham, 1985). Therefore, subjects may occasionally rely on strategies other than retrieval (Siegler, 1988). For instance, the order of the operands may be reversed ($2+9 = 9+2 = 11$), or the problem may be decomposed into simpler memorized facts (e.g. $9\times8 = 10\times8-8 = 80-8 = 72$). It is obvious that such recoding strategies require an understanding of the quantities involved in the original problem (e.g. noticing that 9 is larger than 2 or that 9 is close to 10). This is why Dehaene and Cohen (1995) regroup them under the generic term of "semantic elaboration". Semantic elaboration may be needed both before and after fact retrieval in order to assess the quantities of the operands or to check the plausibility of the retrieved result.

The parietal problem-size effect found in the present study seems quite compatible with this view. Simple problems require little or no semantic elaboration and therefore activate the magnitude representation held in inferior parietal areas rapidly and for only a short period of time. The finding that simple problems activated the inferior parietal cortex at all does not contradict the assumption of the model that the result for simple problems is retrieved from rote verbal memory. Simple problems could have accessed the magnitude representation automatically, although it is not relevant for the retrieval process per se. Several experimental results have indeed suggested that a magnitude representation is automatically activated by numbers even if it is not relevant for task performance (for a recent review, see Dehaene & Akhavein, 1995).

Therefore, it is possible that the inferior parietal effect for simple problems found in the present experiment reflects an automatic activation of a magnitude representation which decays after about 300 msec. It is also possible that, even with simple problems, the subjects briefly use the magnitude representation to reinterpret the problem—for instance, to reorder the operands (Dehaene & Cohen, 1995; see below).

The assumption of a short-lived activation of a magnitude representation in simple problems is supported by the reaction time results of the present study in which the disparity of the false result affected only the verifications of complex problems, but not those of simple problems. This can be taken as evidence that at the end of the retrieval interval, when the proposed result was presented, a magnitude representation of the multiplication problem was activated only for complex problems, not for simple problems. Compatible with this interpretation, complex problems—which presumably needed intense semantic elaboration strategies—elicited sustained activity in parietal areas. Hence, both electrophysiological and behavioural data suggest that only in complex problems was a magnitude representation activated until the proposed result was presented. Finally, the parietal problem-size effect was similar for visual–arabic and auditory–verbal notations, as expected from Dehaene and Cohen's (1995) hypothesis of a modality-independent magnitude representation in inferior parietal cortex.

Different models of number processing might suggest other explanations for the activation of the inferior parietal cortex in mathematical fact retrieval. According to McCloskey's (1992) model, all numerical tasks, including multiplication fact retrieval, require access to an abstract amodal representation of numbers. Although McCloskey has not made explicit which areas or networks of areas are involved in this representation, the present results would be compatible with it being localized to inferior parietal regions. Another model, Campbell and Clark's (1992) encoding complex model, assumes notation-dependent retrieval processes. However, this model does not rule out the existence of notation-independent processes. Hence, the modality-independent problem-size effect found in this experiment is also compatible with the encoding-complex model, although it was not specifically predicted by it. In the context of the encoding-complex model, it may be noted that our data did show small differences in the time course of parietal activation for visual and auditory operands. Although the topography of the problem-size effect was quite similar for visual and auditory operands, its onset was slightly earlier for auditory than for visual operands (294 vs. 334 msec). Latency differences were also found in the offset of the late centro-parietal problem-size effect. For visual operands it ended at about 1,150 msec, whereas for auditory operands it continued until 1,300 msec. At present, it seems quite difficult to decide whether such differences stem merely from the different duration of identification processes in the auditory and visual modalities, or whether they reflect a modality-specific encoding of arithmetical facts in memory, as Campbell and Clark (1992; Campbell, 1994) would predict.

Effect of Operand Order

Another effect potentially relevant to number-processing models was that operand order affected ERPs only for auditory operands, not for visual operands. Starting at about 290 msec, a bilateral prefrontal and simultaneously temporal activation pattern was found. Hence, the topography of the operand-order effect differed from the topography of the problem-size effect which was over left parietal areas. Together with its earlier onset, it can be concluded that operand order affected the calculation process in a qualitatively different manner to that of the problem-size effect. We speculate that the operand-order effect may be due to a strategy of reordering the operands. Reordering strategies are explicitly taught at school in countries like the United Kingdom, Finland, or China. In multiplication, the smaller operand of one pair is always placed in first position (LeFevre & Liu, submitted). Unfortunately, we did not ask our subjects whether they used any such strategy. Previous studies of multiplication in normal subjects have not reported an operand-order effect. Most, if not all of them, however, have never used an auditory presentation of the operands, which is the only condition in which we found a significant effect.

How is the operand-order effect specific to the auditory modality compatible with models of number processing? The finding seems most problematic for McCloskey's (1992) modular framework, which asserts that calculation processes should be the same regardless of the modality of input of the problem. In this model, modality-dependent calculation strategies are explicitly denied. The encoding-complex model, contrariwise, states that calculation procedures may vary with input notation or output requirements (Campbell, 1992, 1994; Campbell & Clark, 1992). It could therefore accommodate the results, although it does not offer any explicit explanation for them. Finally, the triple-code model (Dehaene, 1992; Dehaene & Cohen, 1995) may fare better since it explicitly supposes that multiplication facts are stored in memory in verbal format as they were learnt by rote at school. It seems plausible that, as a result of rote learning, some multiplication facts may be stored only in a given operand order, for instance with the smallest operand first. As a consequence, the sentence /six times nine is fifty-four/ would be stored in verbal memory, but the sentence /nine times six is fifty-four/ would not. When presented with the problem 9×6, subjects would first have to reorder it to 6×9 before being able to retrieve the result from memory.

Why should there be a detectable operand-order effect only when the operands are presented in the auditory modality? According to the triple-code model, the sequence of operations necessary to reorder auditory operands is the following: (1) compare the operands, (2) if the first digit is larger, then reverse the order of the operands using auditory-verbal short-term memory, otherwise leave them in the original order, (3) access rote verbal memory. Hence, when presented with wrongly ordered auditory operands, an additional step of re-ordering (step 2) would be needed which involves auditory-verbal short-term memory. In fact, the topography of the operand-order effect with a temporal and prefrontal scalp distribution is quite compatible with the assumption of increased load on verbal short-term memory due to the reordering of the operands. Paulesu, Frith, and Frackowiak (1993), for instance, found frontal as well as temporo-parietal activations during a verbal working-memory task.

With visual operands, on the other hand, the sequence of operations predicted by the triple-code model is the following: (1) compare the operands, (2) convert the smallest operand to a verbal representation, (3) convert the largest operand to a verbal representation, (4) access rote verbal memory. In contrast to auditory operands, with visual operands the mental operations would be basically the same regardless of the order of the operands. As a consequence, a reordering cost might not be noticeable in the visual format because, according to the triple-code mode, the operands always have to be transcoded into a verbal representation regardless of their initial order. Therefore, auditory-verbal short-term memory would not be engaged by the reordering process in the visual modality. The comparison of the operands, on the other hand, which precedes the reordering operation, would not produce any detectable ERP difference between "smaller operand first" and "larger operand first" trials, because comparison is supposedly common to these two kinds of trials. In summary, the hypothesis of a reordering and of a verbal encoding of the operands of a multiplication problem seems to provide a coherent account of the operand-order effect, its temporo/prefrontal topography, and its modulation by the modality of the operands.

Effect of Result Modality

The expected modality of the proposed result had an early effect starting at about 250 msec after the second operand on temporo-parietal electrodes and at about 540 msec to 850 msec over left inferior frontal electrodes, as well as a late bilateral superior frontal effect starting at about 1,100 msec. With respect to the early effect, the modality of the expected result notation affected ERPs differently depending on the operand notation. For visual operands, a steeper rise of the left parietal positivity was obtained if the result was also presented visually. For auditory operands, an effect of the modality of the proposed result was found only on right parietal electrodes.

The fact that some of these effects were so early suggests that slightly different processing strategies were adopted depending on the upcoming modality of the proposed result. However, the significance of these effects remains unclear and deserves further investigation. The late effect on left inferior frontal electrodes is perhaps associated with an activation of Broca's area if the expected result is auditory. Subjects may then choose to hold the retrieved fact in auditory verbal working memory (Paulesu et al., 1993). The late frontal differences in the ramping negative wave may reflect different states of expectancy which are imposed by the expected result modality. Interestingly, for both visual and auditory operands the negativity was increased on frontal electrode sites if the expected result was presented visually. Similar slow negative waves are frequently observed in paradigms where, following a stimulus S1, subjects develop an expectation for a second stimulus S2 (Bierbaumer et al., 1990; Rösler, 1990; Rösler & Heil, 1991). These negativities are known to vary with the modality of the expected stimulus (Bierbaumer et al., 1990). However, an expected visual stimulus is usually associated with an increased negativity over occipital sites, presumably indexing a preactivation of the visual cortices. In contrast, the present effect was observed mostly on superior frontal electrodes, perhaps reflecting the activation of a working-memory circuit in prefrontal cortex (Jonides et al., 1993; McCarthy et al., 1994).

CONCLUSION

The present study focused mainly on two simple questions. Can an electrical signature of the activation of inferior parietal areas during calculation be observed in scalp recordings of event-related potentials, and can ERPs be used to provide information about the time course of this activation? Our results provide a positive answer. They suggest that simple multiplication problems are processed rapidly by the left parietal cortex, whereas complex problems may require longer processing which also involves the homologous area in the right hemisphere. Furthermore, they indicate that parietal areas are active regardless of the modality of input of multiplication problems. Further research should attempt to characterize in greater detail the nature of the representation of numbers which is manipulated in inferior parietal cortex.

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APPENDIX A

List of the Problems

	Proposed result		
Problem	*true*	*close*	*far*
Simple			
2 × 3	6	10	18
2 × 4	8	10	36
2 × 5	10	14	30
3 × 4	12	16	28
3 × 5	15	25	45
4 × 5	20	16	36
Complex			
6 × 7	42	54	12
6 × 8	48	54	24
6 × 9	54	72	18
7 × 8	56	42	24
7 × 9	63	45	21
8 × 9	72	64	36