The foundations of numerical thinking in a brain without numbers

Tony J. Simon

In a recent paper, Dehaene *et al.*[1] investigated the cognitive and neural bases of mathematical thinking, the key findings of which were summarized by Butterworth as indicating that 'mathematical ability... results from the integration of two non-numerical neural circuits in the brain'[2]. One of these non-numerical circuits is in the left frontal lobe, which is associated with linguistic representations, in this case, representations of exact numerical values. The other is found bilaterally in the parietal lobes, a part of the brain associated with visuospatial functions in general, and, by Dehaene *et al.* in particular, with representations of approximate quantities in the form of a number line[3].

These findings are compelling and provocative and they provide further support for the view that humans have at least two means of representing and processing quantity. One is the ability to make perceptually based judgments and comparisons. In this, degree of accuracy varies with set size. The other allows precise quantification through the use of symbols, concepts and rules. While Dehaene *et al.* address the sources of these different abilities in terms of the adult human brain, they leave untouched the issue of how such functional organization arises. Put another way, their findings beg the question of whether these neural sources of adult mathematical thinking are found in a brain designed for numerical representation and processing, or in one that invents such functions when faced with particular task demands. In what follows I shall suggest (in line with Butterworth's statement) that the latter case is the more plausible. This requires us to ask how the mathematical representations and processes that Dehaene *et al.*'s data point to emerge from an initially non-numerical brain.

It is important to note that evidence linking numerical processing to neural structures in humans has, perhaps unsurprisingly, been restricted to cortical sites[1,4–6]. Cerebral cortex is a part of the human brain which, though structurally complete at birth, is to a considerable extent functionally, and thus representationally, undetermined. Current estimates suggest that no more than 25% of cortex is functionally connected initially, and the immense postnatal changes in density and connectivity indicate almost open-ended plasticity. Despite intense debate about the nature and extent of environmental influence on cortical function[7–9], the fact that some such shaping does occur makes hard-wired representational systems as complex as that required by number quite untenable. More plausibly, to quote Thatcher, numerical competence is constructed via, 'iterative growth spurts and patterns of development during the postnatal period... slowly sculpting and shaping the brain's microanatomy to eventually meet the demands and requirements of an adult world'[8].

Where then might we look to find the foundations of numerical processing in a brain without numbers? Clearly, Dehaene *et al.*'s frontal-lobe 'exact' numerical circuit is an unlikely candidate: infants do not have formal language or number symbols, they are incapable of symbolic arithmetic, and their frontal lobes are functionally underdeveloped. However, the apparent implementation of 'approximate' numerical representation and processing by visuospatial areas does converge with an existing developmental account. I have suggested that numerically relevant competencies demonstrated by infants lie predominantly in their object representation and individuation abilities, which also appear to depend on the brain's visuospatial processing regions[10]. In what follows, I shall outline a developmental trajectory whereby the simplest functions available to infants are handled by visual areas that develop very early. More complex and developmentally advanced functions then rely on visuospatial areas, including those identified by Dehaene *et al.*

Linking enumeration and attention

There is now little doubt that infants are capable of forming representations of simple physical objects, that they can maintain expectations for a short time about the existence of those objects when hidden by a screen, and that they can detect violations of their expectations when objects have 'magically' appeared or disappeared during the time they were not visible[11]. I have proposed an account whereby the observed reactions of infants can be generated from a set of domain-general competencies, each documented in infancy, which together are co-opted by task requirements into behaviors that superficially resemble numerical reasoning[10,12]. At the heart of this set is the ability to individuate, or create unique markers for, up to three or four entities. Infants evidence this competence by being able to make simple discriminations between sets of up to four entities[13]. Children and adults exhibit the same processing in their almost effortless ability to subitize, or quickly apprehend quantity, in collections of up to four items.

The subitizing phenomenon is manifested when individuals quantify visually presented entities as quickly and accurately as possible. The same discontinuity always emerges: up to three or four objects are quantified rapidly and without error, with each extra object adding around 50 ms to reaction time. Larger collections require at least 300 ms for each new item, and errors appear. Trick and Pylyshyn linked this discontinuity with the so-called 'preattentive/attentive' dichotomy in spatial attention[14]. Subitizing was possible when targets were distinct from other entities in the visual field (e.g. when letter Os 'popped out' from a field of letter Xs). Dramatically, the authors transformed the characteristic subitizing/counting reaction-time 'elbow' into a linear slope when the same targets required effortful disambiguation from the distractors (e.g. when the distractor Xs were replaced by an unchanged number of letter Qs). That created a standard conjunctive visual search task that engenders serial individuation. Yantis' finding that attention can be automatically captured by more than one but not more than four abrupt onset stimuli[15] further links individuation of small collections to preattentive visuospatial processing.

Direct neural evidence connecting different modes of attention and enumeration comes from a recent study carried out by my colleagues and myself[6]. Using PET imaging we showed that during the task of subitizing, a site in the right middle/inferior occipital gyrus was activated (Talairach coordinates: 28, –87, 4), consistent with the involvement of early, preattentive visual processing. By contrast, during counting, widespread areas were activated, the most relevant to this discussion being bilateral foci in the superior parietal gyri/intraparietal sulci (Talairach coordinates: –28, –56, 43 and 31, –57, 43). These are areas of cortex that appear to play a central role in

…mon is at the …roscience Center, …nal Institute on …Abuse, NIH, …da, MD …USA.

…301 443 4100
…301 443 1801
…tjsimon@…

1364-6613/99/$ – see front matter PII: S1364-6613(99)01383-2

the shifting of spatial attention[16]. Our parietal foci are broadly similar to those found by Dehaene *et al.*, though they probably cannot be considered to indicate common loci [in both studies the activation regions are quite large, and the Talairach coordinates indicate only the peaks of those areas. Peaks of our sites differed by 18 mm (left) and 20.5 mm (right) from those of Dehaene *et al.*, which is on the boundary of differences considered to discriminate between common and distinct loci]. Given the task differences, this is not very surprising. While both tasks required the determination of the magnitude of a quantity, our task involved summing a set of visible objects while Dehaene *et al.* required their participants to evaluate the result of a calculation based on applying rules to a pair of symbols. However, that two very different studies should activate such similar sites in parietal cortex in response to distinct quantification tasks does suggest that this brain region is actively involved when adults carry out some kinds of numerical computation.

From object individuation to mathematical thinking via counting?

The results of the Dehaene *et al.* and Sathian *et al.* studies discussed above lend support to my hypothesis that the foundations of numerical processing '[emerge] primarily from some general characteristics of the human perception and attention system'[10], that is, in brain regions primarily adapted for visuospatial processing. However, these results allow us to go further, in hinting at the developmental trajectory from simple individuation to mathematical computation. I have already linked the basis of infants' abilities in the magical appearance/disappearance tasks to object individuation. Sathian *et al.*[6] showed that those individuation processes are implemented by an area of extrastriate occipital cortex that is almost certainly functioning within the first few months of life[17]. The same study strengthens the case for the involvement of visuospatial processing areas of adult human cortex in more complex quantification tasks. Areas activated by counting were very close to those associated with approximate calculation, hypothesized to be based on neurally represented number lines. The critical, though admittedly speculative, link from occipital to parietal cortex might lie in the development of counting competence.

It has been shown in recent years that children learn to use the procedures of counting before they acquire the underlying quantitative principles, such as ordinality[18]: knowledge which is itself a prerequisite for arithmetical competence. Consistent with Dehaene's claim[1] that the association between numbers and space is constructed by exposure to cultural conventions, counting objects arranged in physical space probably provides critical early contributions to the construction of the hypothesized number line representation. One convention that pervades childhood, apparently irrespective of culture, is using the fingers to count before the process can be carried out mentally. Children need to make counting behavior concrete and external while the procedure is being practiced. In counting with their fingers they are probably beginning the association between numbers and the parietal lobes, now implicated by Sathian *et al.* in that very process in adulthood! As Butterworth points out, that same area of cortex is 'part of a neural circuit that controls handshapes and finger movements. This raises the possibility that these brain regions contribute to finger counting'[2]. It is striking to note that left parietal damage can result in Gerstmann's syndrome, a complex that includes both the inability to identify fingers touched by another or indicate specific fingers on verbal command (finger agnosia), and the inability to comprehend and process numbers (acalculia).

Two studies linking neural damage with compromised numerical competence appear to support the developmental trajectory I have proposed. In one, a patient with left parietal damage resulting in Gerstmann's syndrome exhibited dense acalculia but 'showed a largely preserved ability to deal with numbers below 4' (Ref. 4). In another, patients with parietal damage exhibited simultanagnosia, 'a deficit of the visual perception of complex scenes, with a preserved recognition of individual objects'[5]. These patients were able to enumerate displays of up to three objects via subitizing, but were unable to count objects in larger displays. In both cases, parietal damage compromised counting and large-number quantification but left untouched the subitizing range, within which infants can also operate. Such evidence supports the view that early visual-processing areas in occipital cortex are responsible for simple object individuation, while more advanced quantitative tasks involving at least the ability to count, are implemented by spatial-processing circuits in the parietal lobes.

Conclusion

Clearly, much more work needs to be done to establish firmly the neural substrates of quantification processing in visuospatial circuitry, but it already looks like a promising enterprise. Dehaene *et al.* have presented data that provide strong new evidence for the dichotomy between perceptually and conceptually based quantitative competence. However, the findings pose new questions about which common cultural experiences are necessary to stimulate our brains to organize themselves in such a way as to add these, initially unspecified, functional capabilities. The fact that societies still exist without requirements to go beyond a number system of three items[19], indicates that Dehaene *et al.*'s findings characterize a pattern of ontogenetic brain organization that has responded to ecological demands. Questions of how environmental stimulation transforms cognitive competence from one level to another are central to the field of developmental cognitive science. Unfortunately, these are not the kinds of investigations that many of us are experienced at tackling. With the powerful new tools of brain imaging to add to our experimental methods we have reached the point where we should be able to discover how mathematical thinking arises from a brain without numbers. These new findings of Dehaene *et al.* and Sathian *et al.* should stimulate us to do so with renewed zeal.

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Biological foundations of numerical thinking

Response to T.J. Simon (1999)

Elizabeth Spelke and Stanislas Dehaene

We are indebted to Tony Simon for raising discussion of the foundations of number processing [Simon, T.J. (1999) The foundations of numerical thinking in a brain without numbers *Trends Cognit. Sci.* 3, 363–364][1]. Simon's remarks bear on two theoretical issues. First, what is the specificity of the cerebral circuits for number processing? Second, how do numerical abilities emerge in the course of development? Simon's answers are clear-cut: the brain's cerebral circuits are 'non-numerical', and the developmental foundations of numerical processing are to be found in 'a brain without numbers' which constructs itself through unspecified mechanisms of 'open-ended plasticity'. We disagree on all counts. Although those important issues are still open to scientific inquiry, there is already strong evidence that the numerical abilities of the human brain rest in part on specialized cerebral processes and follow a specific developmental time course that hints at an initial specialization.

Specialized cerebral circuits for number processing

Our behavioral and brain-imaging results indicate that the rote learning of arithmetic tables is based on a linguistic representation of numbers, and therefore that such learning requires the 'recycling' of initially non-numerical brain circuits, such as language circuits, for the purpose of mathematics[2]. However, is that the case for *all* of our mathematical competences? Our research has uncovered a second cerebral circuit that depends on the left and right intraparietal regions, underlies the understanding of proximity relations between numerical quantities, and is particularly important for approximation and number comparison. We view this circuit as providing a biological foundation for number sense.

Three types of findings support the notion that the inferior parietal cortices contribute to a biologically determined numerical representation. First, left inferior parietal lesions can specifically impair the understanding and processing of numbers[3,4]. Conversely, the category of number can also be selectively preserved in the presence of severe deficits in the processing of other categories of words[5]. The dissociation between preserved and impaired knowledge can be remarkably sharp. For example, patient 'MAR' was at chance in deciding which number falls between two others (he responded that the number that falls between 1 and 3 was 7), yet he had no trouble performing similar bisection tasks with letters, months, days of the week or notes of the musical scale. Such category-specific deficits exist for other domains as well. For instance, a remarkably restricted impairment of the knowledge of animals was recently described[6]. Based on such cases, the suggestion has been made that, through brain evolution, specialized regions of the brain have emerged for the representation and manipulation of evolutionarily relevant environmental categories, such as animals, persons, or foods[2,6]. Number also appears to be such a category.

A second finding comes from cross-cultural comparisons of the cerebral bases of calculation, which provide evidence that the association of arithmetical function with the intraparietal sulcus is remarkably reproducible. If arithmetic were just a cultural activity without a strong foundation in brain architecture, one would expect considerable variation, depending on learning, education and culture. Yet reports from research groups in various countries suggest that, in most if not all cultures throughout the world, the sites of the lesions causing a loss of number sense, as well as the sites of brain activation during calculation, systematically fall in the inferior parietal region (see Ref. 7 for a review).

The third source of evidence that speaks against Simon's notion of 'open-ended plasticity' comes from studies of developmental dyscalculia, which indicate that the contribution of the inferior parietal cortices to number processing is highly specific and cannot be easily transferred to other brain regions. Some children show a selective category-specific deficit for number processing[8,9]. In spite of their normal intelligence, normal language acquisition, and the special education that they received, they were never able to acquire the concept of number. They have to rely on laborious verbal-counting strategies even for tasks as simple as determining that nine is larger than three, or that a duck has two legs. Although few accurate brain-imaging data are available on patients with developmental dyscalculia, in at least one case the deficit has been related to early brain damage restricted to a small region of left inferior parietal cortex[10]. Thus, the parietal circuit would appear to be functional during development to such an extent that its lesioning causes a complete failure of arithmetic development. Such evidence is hard to reconcile with the idea, implicit in Simon's comment, that the brain is a general learning device. There is, of course, no denying that learning occurs in the mathematical domain. But learning might be based on specialized domain-specific systems rather than on a general constructivist scheme.

Note that the postulated role of parietal circuitry in providing a biologically determined sense of number does not imply that arithmetic is the *only* function of that circuit. It is naive to expect current brain-imaging techniques to reveal a single portion of brain tissue that is responsive only to numbers. Rather, the extent of our observed activations hints at the possibility of a considerable overlap with other visuospatial functions that are known to yield very similar activity patterns in the intraparietal sulcus, such as mental rotation and other spatial-coordinate-transformation tasks. Simulations in neural networks suggest that a representation of numerical quantity can emerge naturally from the extraction of object-location information, independently of object identity[11]. Because these functions are performed within the dorsal occipito-parietal pathway, parietal circuitry might have needed only minimal alteration in the course of evolution to become biased to encode numerosity information. Numerical and spatial representations might thus be intricately intertwined in the parietal lobe. It is possible that they can only be distinguished empirically by their internal micro-circuitry or their pattern of connectivity to other brain areas.

Phylogeny and ontogeny of numerical representations

Because our article was concerned only with the performance of human adults, it does not in itself address the questions at the heart of Simon's essay, which concern the phylogeny and ontogeny of numerical representations. There is,

E. [illegible]pelke is at the Department of Brain and Cognitive Sciences, Massachusetts Institute of Technology, Cambridge, MA [illegible]2139, USA.

tel: [illegible] 617 253 7957
fax: [illegible] 617 258 8654
e-mail: spelke@mit.edu

S. [illegible]aene is at the [illegible] INSERM 334, S[illegible] Hospitalier F[illegible] Joliot, CE[illegible] SV, 91401 [illegible] Cedex, France.

tel: [illegible]9 86 78 73
fax: [illegible]9 86 78 16
e-mail: [illegible]shfj.cea.fr

however, a wealth of animal and developmental research that is pertinent to these topics, and much of that research also fails to support Simon's thesis of 'a brain without numbers'.

Let us begin with phylogeny. If evolution did not produce a brain with the capacity to represent number, and if that capacity emerged in humans only through culturally transmitted activities unique to us, such as finger counting, then other animals should show no capacity to represent numerosity. Contrary to this prediction, a wealth of research by behavioral ecologists and comparative psychologists provides evidence for capacities to represent numerosity in animals including birds, rodents, and primates (for reviews, see Refs 2,12). Abilities to discriminate between sets of different numbers of items, and to base that discrimination on number rather than on other perceptual variables such as spatial extent or temporal duration, have been found in numerous experiments. For example, laboratory rats, parrots and monkeys have been trained to respond to a specific number of objects or events[13–15], and untrained monkeys and apes have been found to choose spontaneously the more numerous of two sets of food items[16,17]. Because no non-human animal (with the possible exception of language-trained chimpanzees) has been observed to engage in finger-counting, these findings fail to support the claim that the sense of number results from such activities. More positively, the findings provide evidence that a capacity to represent number evolved before humans did and is shared by many vertebrates.

Turning to ontogeny, we can ask whether Simon's account nevertheless is true of humans: do human infants represent and track up to four objects, but otherwise fail to represent sets and larger numbers? It is possible that representations of objects, rather than explicit representations of numerosity, underlie infants' performance in Wynn's addition and subtraction tasks[18] and in number-discrimination tasks that present small numbers of objects[19]. Such representations cannot, however, account for infants' performance in a variety of other situations. Experiments provide evidence that young infants can enumerate entities that are not material objects, including speech sounds[20] and actions such as jumping[21]. Moreover, infants can distinguish between sets whose numerosities exceed the limits on object representations: preliminary evidence suggests that they can distinguish displays of 8 versus 16 dots when variables such as spatial extent, density and brightness are controlled (F. Xu and E. Spelke, unpublished data). All these findings provide evidence for representations of number that exceed the scope and power of mechanisms of object tracking (see Ref. 22 for a review).

Despite the evidence for phylogenetic and ontogenetic continuity in number representations, human number representations do have unique features. In particular, only humans who have learned symbolic counting appear to represent the exact numerosities of sets with no upper limit. When animals are trained to make exact-number discriminations, training becomes increasingly difficult, and performance increasingly error prone, with increasing numerosity[15,23]. In the absence of such training, discriminability is proportional to set size, in accordance with the Weber–Fechner law[12]. Human infants also discriminate between large numerosities only when the difference ratio also is large: infants have been found successfully to distinguish 8 from 16 dots but not 8 from 12 dots (F. Xu and E. Spelke, unpublished data). Once children learn verbal counting, however, they come spontaneously to use counting to represent large numerosities exactly[24,25]. It is possible that the involvement of language areas of the brain in the memorization of exact arithmetic facts is rooted in this acquisition.

In summary, we share with Simon the hypothesis that some mathematical abilities, particularly those that are evidently late cultural acquisitions, such as multiplication tables, do not rely on specific cerebral substrates. The unique and culture-specific features of human number knowledge nevertheless appear to build on a dedicated neural and cognitive system: a number sense that emerged early in vertebrate evolution, is present and functional early in human development, and resides in dedicated neural circuitry.

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Comment coming soon...

Theta activity, virtual navigation and the human hippocampus, by John O'Keefe and Neil Burgess

Response from Michael J. Kahana, Robert Sekuler, Jeremy B. Caplan, Matthew Kirschen and Joseph R. Madsen