Introduction

Special Issue: Space, Time and Number

Space, time, and number: a Kantian research program

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'The knowledge of first principles, as space, time, motion, number, is as sure as any of those which we get from reasoning. And reason must trust these intuitions of the heart, and must base on them every argument.'

Blaise Pascal, *Pensées* (translated by W. F. Trotter)

What do the representations of space, time and number share that might justify their joint presence in a special issue of *TICS*? In his *Critique of Pure Reason*, Immanuel Kant famously argued that they provide '*a priori* intuitions' that precede and structure how humans experience the environment. Indeed, these concepts are so basic to any understanding of the external world that it is hard to imagine how any animal species could survive without having mechanisms for spatial navigation, temporal orienting (e.g. time-stamped memories) and elementary numerical computations (e.g. choosing the food patch with the largest expected return) [1]. In the course of their evolution, humans and many other animal species might have internalized basic codes and operations that are isomorphic to the physical and arithmetic laws that govern the interaction of objects in the external world [2]. The articles in this special issue all support this point of view: from grid cells to number neurons, the richness and variety of mechanisms by which animals and humans, including infants, can represent the dimensions of space, time and number is bewildering and suggests evolutionary processes and neural mechanisms by which Kantian intuitions might universally arise.

Space, time and number also raise deep computational issues for cognitive neuroscience. In all three domains, the nervous system must encode and compute with quantities. Behavioral evidence suggests that these computations can be remarkably accurate, even in miniature organisms, such as desert ants, or in immature systems, such as the human infant brain. Animal spatial navigation implies a mental storage of spatial coordinates and their updating by path integration [1]. Temporal decisions imply that memorized representations of time are subjected to operations that are analogous to addition, subtraction and comparison [3]. In the number domain, preverbal organisms such as human infants and many non-human animal species readily anticipate the outcome of analogs of arithmetic operations performed with concrete sets of objects [4]. Does this mean that a common set of coding and computation mechanisms underlie quantity manipulations in all three domains? Do they share similar brain circuitry? An exciting research program exists that involves mapping the range of possible implementations of quantitative operations in the nervous system, and testing whether evolution has arrived at the same computational solutions in distinct organisms or for distinct domains.

The 24th Attention & Performance meeting on 'Space, Time, and Number: Cerebral Foundations of Mathematical Intuitions', held on 6–10 July 2010 in Vaux de Cernay, near Paris, was organized with this goal in mind: to clarify the fundamental points of convergence and divergence between the representation of number, space and time. The six papers that appear in this special issue were chosen to represent the 24 talks that were presented, and which will appear as chapters in an edited book [5]. As the meeting unfolded, it became clear that, although remarkable progress has been made in mapping out the behavioral competence, brain areas and sometimes the single-cell mechanisms underlying specific spatial, temporal or numerical tasks, the domain in its full generality remains unsystematically explored. A general research program lies ahead, which could be aptly called a 'Kantian' research program, as it aims to understand how basic intuitions arise, how they can be related to their neural mechanisms, which aspects of these mechanisms arise independently of experience, and which can be enriched by training and education.

Rather than attempting to summarize the diversity of insights and discoveries that have been made in this field, and which are well represented by the articles in this special issue, here we highlight the range of questions that we believe should be part of this Kantian research program and, indeed, can be productively addressed using present-day cognitive neuroscience methods.

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Quantity codes

How are quantities, such as spatial coordinates, distances, times, durations or numbers, encoded in the nervous system? Electrophysiology using animal models has uncovered mechanisms that include neurons tuned to specific values; neurons with monotonically increasing or decreasing firing as a function of quantity; neurons with periodical firing patterns, such as grid cells; and neurons with a rich diversity of dynamics capable of generating a unique, partially random collective code for any quantity (see Buonomano and Laje, this issue). Do these mechanisms exhaust the range of possibilities? How do these mechanisms account for the many orders of magnitudes that can be simultaneously represented in the brain, for instance in the time domain? And can one extrapolate from animal models to the human neural code for space, time and number?

Developmental origins

Are neural codes for space, time and number available early during development, so that they can have a determining role in structuring subsequent experience, as postulated by Kant? Or are they, by contrast, extracted by learning mechanisms following exposure to a richly structured physical world? The hypothesis that 'innate' mechanisms underlie spatial orientation mechanisms has received a major boost lately with the finding, by two independent groups, that several (but by no means all) aspects of the underlying neural machinery of head direction cells, grid cells and place cells are already in place in newly born rats before any significant navigation experience (see Derdikman and Moser, this issue). This research is energizing a renewed research program focusing on the search for representations of space, time and number inherited from evolution (see Haun *et al.*, this issue). However, we acknowledge that the word 'innate', meaning 'independent of experience', is an idealization that must be ultimately replaced by detailed research into the underlying genetic and developmental mechanisms.

Cross-dimensional interactions and metaphors

Do the representations of space, time and number share neural resources (see Burr *et al.*, this issue)? Or do they interact through systematic cross-dimensional mappings? Does a generalized sense of 'magnitude' underlie them all, as recently suggested by the discovery that human infants spontaneously link the dimensions of size, numerosity and duration [6,7]? Or is a single dimension, for instance space, used as a reference for all the others, as is suggested by the observation that, in human languages, spatial terms are frequently used metaphorically to refer to time and number?

Quantitative computations

What are the brain mechanisms by which basic arithmetic operations are implemented? At a minimum, behavioral research indicates that operations of larger–smaller comparison, addition and subtraction must be available in all three domains of space, time and number. Multiplication and division are also frequently needed, for instance to compute speed (space divided by time) or rate of return (number divided by time). Are these operations always implemented by ad hoc evolved devices (e.g., neurons in the motion-sensitive middle temporal area MT/V5 that act as motion filters; or neurons in the lateral intraparietal region that multiplex eye position and retinal position)? Or is there a more general and shared brain machinery by which all quantities, regardless of their domain of origin, can be operated upon?

Thought with or without symbols

Are quantity computations implemented by analog devices? Or do digital or symbolic coding devices exist, even in non-human species? How does one move from approximate computations in animals to exact truth values in human mathematics? How is the neural code for number specifically changed in humans by the acquisition of cultural symbols, such as Arabic numerals (for two different perspectives on this question, see Piazza, this issue, and Butterworth, this issue)? Do symbolic computations 'recycle' evolutionarily older mechanisms for non-symbolic quantity processing [8]?

Human Turing machine

In humans at least, quantities can enter into sophisticated multi-step calculation and decision algorithms that can be likened to computer programs. Do these computations imply specifically human brain mechanisms that grant humans the computational power of a Turing machine? Does the human brain contain dedicated mechanisms for the necessary operations of 'routing' [9] (selecting one out of many input–output mappings), 'chaining' [10] (reusing the output of a process as the input to another), 'if–then' branching, or 'for' and 'while' loops? Can multistep operations unfold automatically or are they necessarily under conscious control?

Impact of culture and education

Education to formal mathematical concepts can enhance the human ability to reason about number, time and space. This is clearest in the case of number, where many concepts are traceable to a recent cultural invention (e.g. decimal numbers, zero, fractions and negative numbers). Is this also the case for space and time? What is it that changes? Can the concept of 'mathematical intuition', as put forward by many mathematicians, such as Henri Poincaré and Jacques Hadamard, be traced back to pre-experiential Kantian representations already present in other animals, or do human intuitions change as advanced mathematical knowledge is acquired?

None of the papers in this special issue solve these difficult problems. However, they all shed light on how they can be addressed empirically, with a combination of behavioral, neuro-imaging and neurophysiological methods in animals, preverbal infants, children and adults. The development of mathematical and simulation models of quantity processing in the nervous system is progressing at a similarly rapid pace. We are therefore optimistic that the above research program will see significant progress during the next decade. It is also likely to lead to important advances, not only at the conceptual level, but also in practical terms, illuminating the issue of how human education impacts on the brain and how brain sciences can inform educational practices. If Immanuel Kant or Blaise Pascal were born today, they would probably be cognitive neuroscientists!

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